Imperial College London

MODULE 6 & 7 CLASS

Aidan Hogg & Patrick Naylor - Autumn Term 2020
ELEC50013: Signal and Systems
Department of Electrical and Electronic Engineering

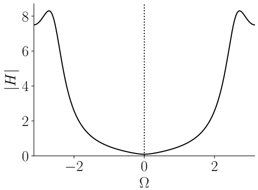
Method:

1: Conceptual question posed - students individually come up initial answer (5 mins)

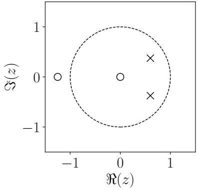
2: Explanation/discussion of correct answer (5 mins)

QUESTION 1:

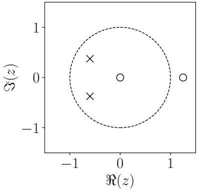
Consider a discrete-time system with the following frequency response



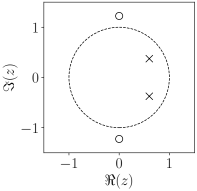
What system gives this frequency response?



A

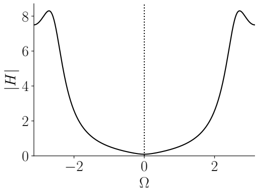


B



C

Consider a discrete-time system with the following frequency response



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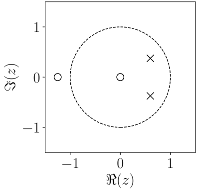
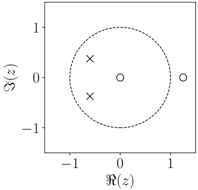
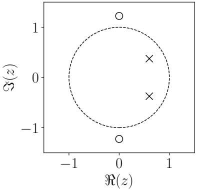


A

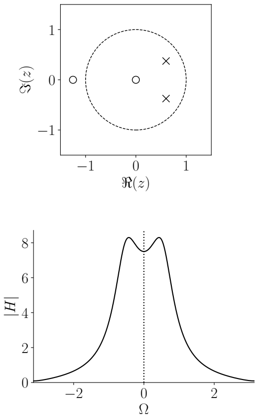


B

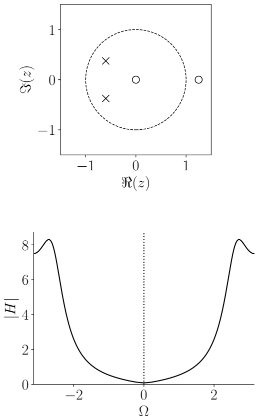


C

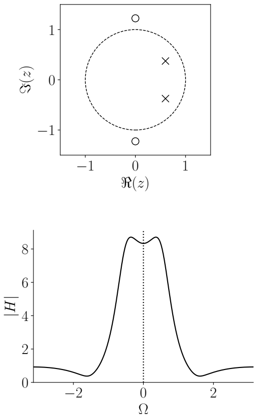
Frequency responses:



A



B



C

QUESTION 2:

Consider an LTI system operating with sampling frequency 10 kHz and system function

$$H(z) = z^{-1}$$

Determine the group delay of $H(z)$

A: 1 Sample

B: 1 Second

C: 1 Radians per second

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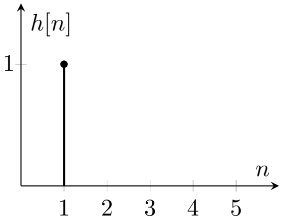
EXPLANATION

One way to solve this: $H(z) = z^{-1} \implies h[n] = \delta[n-1]$



Consequently, the difference equation is:

$$y[n] = x[n-1]$$

Therefore the group delay is 1 sample or 1/10000 = 0.1 ms

EXPLANATION

Another way is to recall that the group delay is defined

$$\tau_H(e^{j\Omega}) = -\frac{d\angle H(e^{j\Omega})}{d\Omega}$$

One trick to get at the phase is:

$$\ln H(e^{j\Omega}) = \ln |H(e^{j\Omega})| + j\angle H(e^{j\Omega})$$

$$H(z) = z^{-1} \implies H(e^{j\Omega}) = e^{-j\Omega} \quad \text{therefore} \quad \ln H(e^{j\Omega}) = -j\Omega$$

thus

$$\angle H(e^{j\Omega}) = -\Omega$$

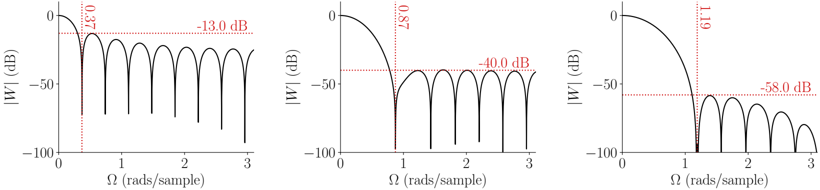
and

$$\tau_H(e^{j\Omega}) = -\frac{d\angle H(e^{j\Omega})}{d\Omega} = 1$$

Therefore the group delay is 1 sample or 1/10000 = 0.1 ms

QUESTION 3:

Consider the following windows:



Rectangular

Hamming

Blackman-Harris

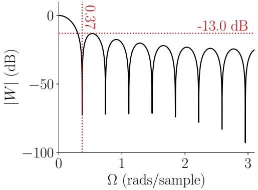
If we were to design a filter using the windowing method, which window should be selected if we only care about the transition bandwidth being narrow?

A: Rectangular

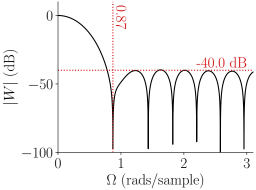
B: Hamming

C: Blackman-Harris

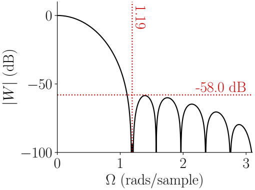
Consider the following windows:



Rectangular



Hamming



Blackman-Harris

If we were to design a filter using the windowing method, which window should be selected if we only care about the transition bandwidth being narrow?

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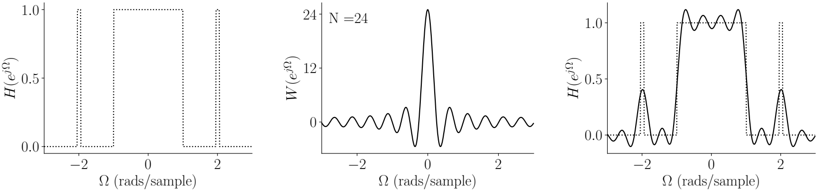
Window Relationships

Relationship when you multiply an impulse response $h[n]$ by a window $w[n]$ that is of length N

$$H_N(e^{j\Omega}) = \frac{1}{2\pi} H(e^{j\Omega}) \circledast W(e^{j\Omega})$$



Transition bandwidth, $\Delta\Omega =$ width of the main lobe

Rectangular window: $\Delta\Omega = \frac{4\pi}{N}$

QUESTION 4:

Recall that a digital system in general is described by a difference equation where a recursive difference equation uses past outputs while a non recursive difference equation does not.

Consider the following statements:

1: FIR filters can only be implemented non recursively in practice

2: IIR filters can only be implemented recursively in practice

Which of these statements are true?

A: 1

B: 2

C: Both 1 and 2

Recall that a digital system in general is described by a difference equation where a recursive difference equation uses past outputs while a non recursive difference equation does not.

Consider the following statements:

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Which of these statements are true?

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EXPLANATION

Consider this FIR filter described by a non recursive difference equation:

$$y[n] = x[n] + x[n-1] + \cdots + x[n-50] \qquad \text{(a)}$$

$$y[n-1] = x[n-1] + x[n-2] + \cdots + x[n-51] \qquad \text{(b)}$$

$$y[n] - y[n-1] = x[n] - x[n-51] \qquad \text{(a) - (b)}$$

$$y[n] = y[n-1] + x[n] - x[n-51] \qquad \textbf{Recursive}$$

(a) FIR filters are normally implemented non recursively but can be implemented recursively.

(b) IIR filters can only be implemented recursively in practice because an infinite number of coefficients would be required to realize them non recursively (recall: $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$).

QUESTION 5:

A Finite Impulse Response (FIR) filter denoted $B(z)$ with input $X(z)$ has output $Y(z) = B(z)X(z)$.

Which of these filters will have linear phase?

A: $b_1[n] = \{1,\ 0,\ 1,\ 0,\ 0,\ 1,\ 0,\ 1\}$

B: $b_2[n] = \{1,\ 1,\ 1,\ 0,\ 0,\ -1,\ -1,\ -1\}$

C: $b_3[n] = \{1,\ 1,\ 0,\ 1,\ 1,\ 1,\ 1,\ 0,\ 1\}$

D: $b_4[n] = \{0,\ 1,\ 0,\ 1,\ 0,\ 0,\ -1,\ 0,\ -1\}$

ANSWER

A Finite Impulse Response (FIR) filter denoted $B(z)$ with input $X(z)$ has output $Y(z) = B(z)X(z)$.

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Linear Phase Filters

The phase of a linear phase filter is: $\angle H(e^{j\Omega}) = \theta_0 - \alpha\Omega$

(where the phase response is a linear function of Ω where α and θ_0 are constant)

Thus the group delay is constant: $\tau_H = -\frac{d\angle H(e^{j\Omega})}{d\Omega} = \alpha$

A filter has linear phase, if and only if, its impulse response $h[n]$ is symmetric or antisymmetric:

$h[n] = h[N-1-n] \quad \forall n$ or else $h[n] = -h[N-1-n] \quad \forall n$

N can be odd ($\implies$ a mid point exists) or even ($\implies$ a mid point does not exist)

Important: This is not the same symmetry that is needed to make the signal real in the frequency domain, which is when $h[n] = h[-n]$.

MERRY CHRISTMAS