Imperial College London

MODULE 5 & 6 CLASS

Aidan Hogg & Patrick Naylor - Autumn Term 2020
ELEC50013: Signal and Systems
Department of Electrical and Electronic Engineering

Method:

1: Conceptual question posed - students individually come up initial answer **(5 mins)**

2: Explanation/discussion of correct answer **(5 mins)**

QUESTION 1:

Suppose we know that if the input to an LTI system is:

$$x(t) = e^{-3t}u(t)$$

then the output is

$$y(t) = [e^{-t} - e^{-2t}]u(t)$$

Which of these statements would be true about this LTI system?

(multiple options allowed)

A: The LTI system is "BIBO stable"

B: The impulse response, $h(t)$, is absolutely integrable

C: The region of absolute convergence of the transfer function, $H(s)$, includes the $j\omega$-axis

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EXPLANATION

Taking Laplace transforms of $x(t)$ and $y(t)$, we get

$$X(s) = \frac{1}{s+3}, \quad \text{ROC: } \Re(s) > -3$$

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2} = \frac{1}{(s+1)(s+2)}, \quad \text{ROC: } \Re(s) > -1$$

Therefore

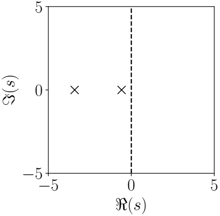
$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+3}{(s+1)(s+2)} = \frac{s+3}{s^2+3s+2}, \quad \text{ROC: } \Re(s) > -1$$

We can now conclude that the system is **stable** (so all the statements are true) and we can even specify the system's differential equation:

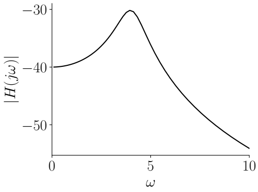
$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + 3x(t)$$

QUESTION 2:

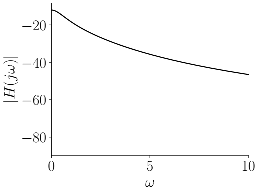
Consider the following pole zero plot in the s-plane



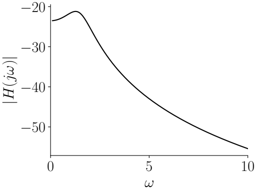
What is the frequency response of this system?



A



B

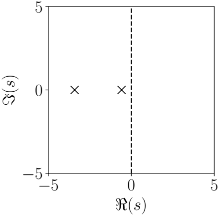


C

ANSWER

[RELATED LECTURE SLIDES: 40- 43 (M5)]

Consider the following pole zero plot in the s-plane



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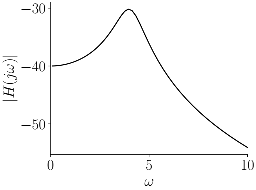
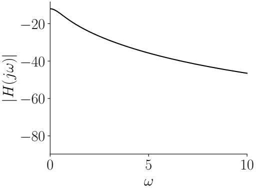
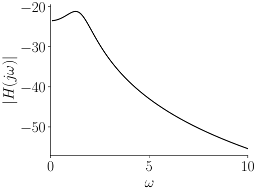


A

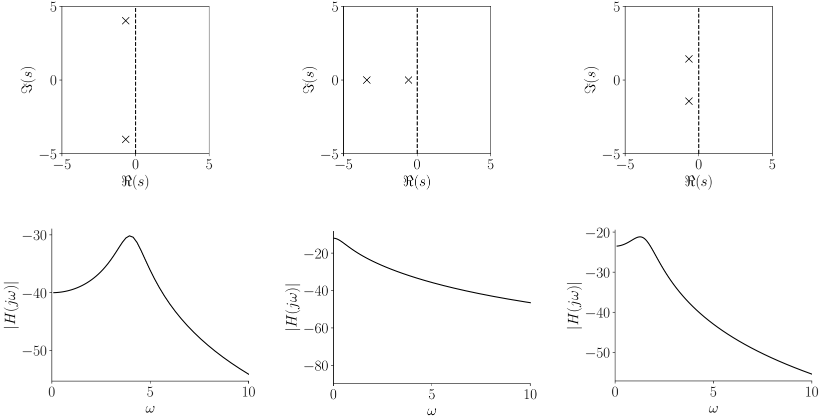


B



C

Frequency responses:



A B C

QUESTION 3:

Consider the following system:

$$y[n] = 2x[n] - 3x[n-1] + x[n-2]$$

Where are the poles and zeros located?

A: Zeros at $z = \{2, 0\}$ and $\{1, 0\}$ Poles at $z = \{0, 0\} \times 2$

B: Zeros at $z = \{2, 0\}$ and $\{1, 0\}$ Pole at $z = \{0, 0\}$

C: Zeros at $z = \{1/2, 0\}$ and $\{1, 0\}$ Poles at $z = \{0, 0\} \times 2$

Consider the following system:

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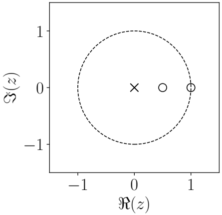
EXPLANATION

Solution

$$y[n] = 2x[n] - 3x[n-1] + x[n-2]$$
$$Y(z) = 2X(z) - 3z^{-1}X(z) + z^{-2}X(z)$$
$$Y(z) = [z^{-2} - 3z^{-1} + 2]X(z)$$
$$Y(z) = [(z^{-1} - 2)(z^{-1} - 1)]X(z)$$

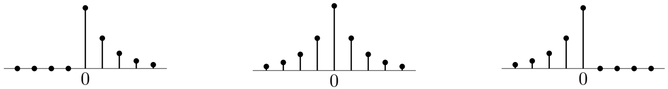


Therefore

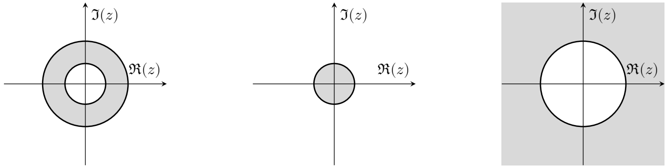
Zeros at $z = \left\{\frac{1}{2}, 0\right\}$ and $\left\{1, 0\right\}$

Poles at $z = \left\{0, 0\right\}$ and $\left\{0, 0\right\}$

QUESTION 4:



(i) (ii) (iii)



(x) (y) (z)

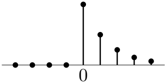
What is the correct mapping between these signals and their ROCs?

A: **(i)** : (x), **(ii)** : (y), **(iii)** : (z)

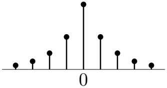
B: **(i)** : (y), **(ii)** : (x), **(iii)** : (z)

C: **(i)** : (z), **(ii)** : (x), **(iii)** : (y)

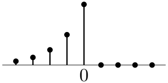
ANSWER [RELATED LECTURE EXERCISES: 2 & 3 (M6)]



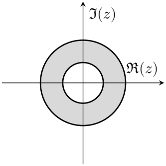
(i)



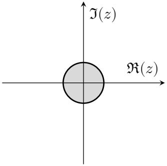
(ii)



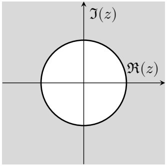
(iii)



(x)



(y)



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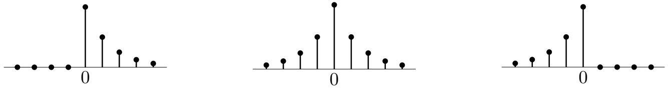
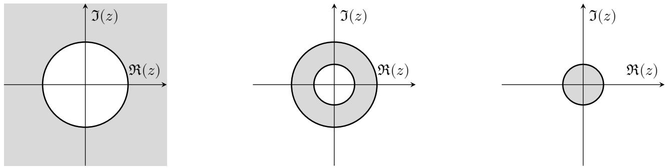
B: **(i)** : (y), **(ii)** : (x), **(iii)** : (z)

C: **(i)** : (z), **(ii)** : (x), **(iii)** : (y)

EXPLANATION



Causal Two-sided Anticausal



If a right-sided impulse response is also causal what does that imply?

If a left-sided impulse response is also anti-causal what does that imply?

QUESTION 5:

Consider the following systems:

$$H(z) = \frac{0.75}{1 + 0.5z^{-1}} + \frac{1.25}{1 - 1.5z^{-1}} \quad \text{ROC: } 1.5 < |z| \leq \infty$$

$$y[n] = 0.5x[n] + 0.5y[n-1]$$

$$h[n] = -1.75^n u[-n-1]$$

What facts are true about all these systems? (multiple options allowed)

A: They are all causal

B: They are all stable

C: They all have an infinite impulse response

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EXPLANATION

Consider the following systems: **Statements True**

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$H(z) = \frac{0.75}{1+0.5z^{-1}} + \frac{1.25}{1-1.5z^{-1}}$ ROC: $1.5 < \lvert z \rvert \leq \infty$	A,C
$y[n] = 0.5x[n] + 0.5y[n-1]$	A,B,C
$h[n] = -1.75^n u[-n-1]$	B,C

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