Imperial College London

MODULE 4 CLASS

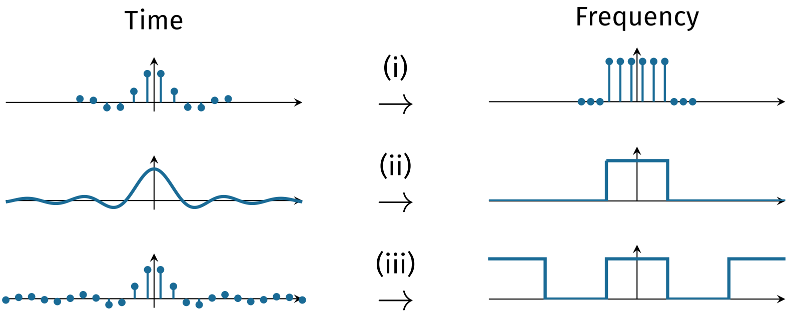
Aidan Hogg & Patrick Naylor - Autumn Term 2020
ELEC50013: Signal and Systems
Department of Electrical and Electronic Engineering

Method:

1: Conceptual question posed - students individually come up initial answer (5 mins)

2: Explanation/discussion of correct answer (5 mins)

QUESTION 1:



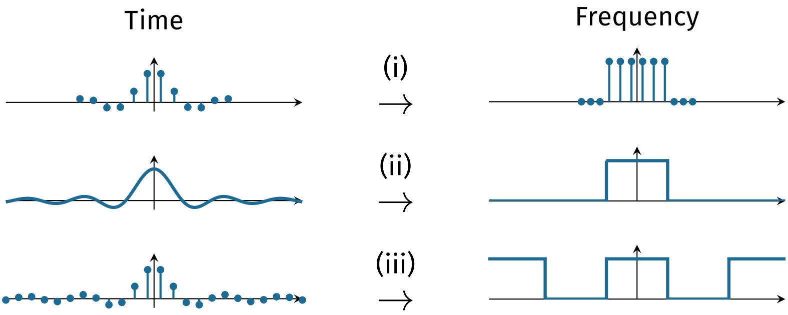
What are the correct names for these 3 transformations?

A: (i)-DTFT, (ii)-CTFT, (iii)-DTFT

B: (i)-DFT, (ii)-DTFT, (ii)-DFT

C: (i)-DTFT, (ii)-DTFT, (iii)-DFT

D: (i)-DFT, (ii)-CTFT, (iii)-DTFT



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D: (i)-DFT, (ii)-CTFT, (iii)-DTFT

QUESTION 2:

Which $x[n]$ will have a purely real $X[k]$?

A: $x[n] = \{0,\ 1,\ 1,\ 0,\ 0,\ 0,\ -1,\ -1\}$

B: $x[n] = \{1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1\}$

C: $x[n] = \{1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 1,\ 1\}$

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B: $x[n] = \{1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1\}$

C: $x[n] = \{1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 1,\ 1\}$

EXPLANATION

Which $x[n]$ will have a purely real $X[k]$?

A: $x[n] = \{0,\ 1,\ 1,\ 0,\ 0,\ 0,\ -1,\ -1\}$ ($X[k]$ will be purely imaginary)

B: $x[n] = \{1,\ 1,\ 0,\ 1,\ 1,\ 0,\ 1,\ 1\}$ ($X[k]$ will be complex)

C: $x[n] = \{1,\ 1,\ 1,\ 0,\ 1,\ 0,\ 1,\ 1\}$ **($X[k]$ will be purely real)**

QUESTION 3:

Compute the Discrete Fourier Transform (DFT) of $p[n] = \{1, 2, 0, 1\}$

What is the value of $P[1]$ and $P[2]$?

A: $P[1] = 4$, $P[2] = 1 + j$

B: $P[1] = 1 - j$, $P[2] = -2$

C: $P[1] = 1 + j$, $P[2] = -2$

D: $P[1] = -2$, $P[2] = 1 + j$

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EXPLANATION

$$P[k] = \sum_{n=0}^{N-1} p[n] e^{-j\frac{2\pi}{N}kn}, \quad k = 0, 1, 2, \cdots N-1$$

Where $N = 4$ and $p[n] = \{1, 2, 0, 1\}$. Note that $e^{-j\frac{\pi}{2}} = -j$

$$k = 0 : P[0] = p[0](-j)^0 + p[1](-j)^0 + p[2](-j)^0 + p[3](-j)^0 = 1 + 2 + 0 + 1 = 4$$

$$k = 1 : P[1] = p[0](-j)^0 + p[1](-j)^1 + p[2](-j)^2 + p[3](-j)^3 = 1 - 2j + 0 + j = 1 - j$$

$$k = 2 : P[2] = p[0](-j)^0 + p[1](-j)^2 + p[2](-j)^4 + p[3](-j)^6 = 1 - 2 + 0 - 1 = -2$$

$$k = 3 : P[3] = p[0](-j)^0 + p[1](-j)^3 + p[2](-j)^6 + p[3](-j)^9 = 1 + 2j + 0 - j = 1 + j$$

$$P[k] = \{4,\ 1 - j,\ -2,\ 1 + j\}$$

EXPLANATION

Here is the DFT matrix of F_N for $N = 4$. Where $w_N = e^{-j\frac{2\pi}{N}}$ (i.e $w_4 = e^{-j\frac{2\pi}{4}} = -j$)

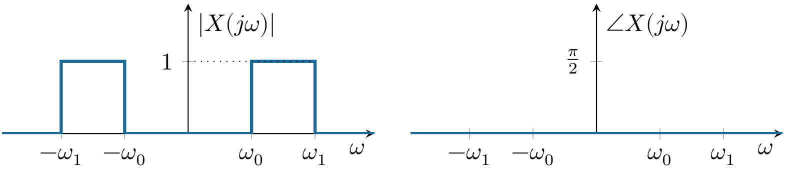
$$\text{DFT Matrix: } F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w_4 & w_4^2 & w_4^3 \\ 1 & w_4^2 & w_4^4 & w_4^6 \\ 1 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Therefore:

$$P[k] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1-j \\ -2 \\ 1+j \end{bmatrix}$$

QUESTION 4:

If $X(j\omega)$ has the following magnitude and phase:



Determine the value of $x(t)$:

A: $\frac{1}{\pi t}\sin(\omega_1 t) - \frac{1}{\pi t}\sin(\omega_0 t)$

B: $\frac{\pi}{\omega_1}\operatorname{sinc}\left(\frac{\omega_1 t}{\pi}\right) - \frac{\pi}{\omega_0}\operatorname{sinc}\left(\frac{\omega_0 t}{\pi}\right)$

C: $\frac{1}{\pi t}\sin(\omega_1 \pi t) - \frac{1}{\pi t}\sin(\omega_0 \pi t)$

D: $\frac{\omega_1 - \omega_0}{\pi}\operatorname{sinc}\left(\frac{\omega_1 t - \omega_0 t}{\pi}\right)$

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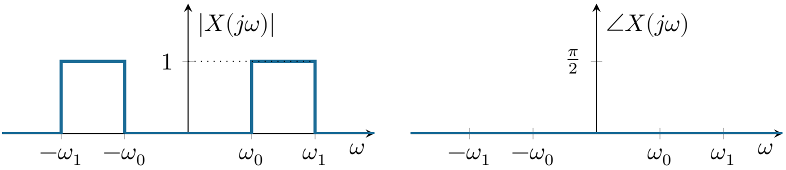


Determine the value of $x(t)$:

A: $\frac{1}{\pi t}\sin(\omega_1 t) - \frac{1}{\pi t}\sin(\omega_0 t)$

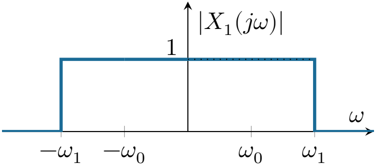
B: $\frac{\pi}{\omega_1}\operatorname{sinc}\left(\frac{\omega_1 t}{\pi}\right) - \frac{\pi}{\omega_0}\operatorname{sinc}\left(\frac{\omega_0 t}{\pi}\right)$

C: $\frac{1}{\pi t}\sin(\omega_1 \pi t) - \frac{1}{\pi t}\sin(\omega_0 \pi t)$

D: $\frac{\omega_1 - \omega_0}{\pi}\operatorname{sinc}\left(\frac{\omega_1 t - \omega_0 t}{\pi}\right)$

EXPLANATION

If $|X_1(j\omega)|$ has the following magnitude:



$$X_1(j\omega) = \begin{cases} 1, & -\omega_1 \leq \omega \leq \omega_1 \\ 0, & |\omega| > \omega_1 \end{cases}$$

$$\begin{aligned} x_1(t) &= \frac{1}{2\pi} \int_{-\omega_1}^{\omega_1} e^{j\omega t} d\omega \\ &= \frac{1}{2j\pi t} \left[e^{j\omega t} \right]_{-\omega_1}^{\omega_1} \\ &= \frac{1}{\pi t} \sin(\omega_1 t), \quad t \neq 0 \end{aligned}$$

EXPLANATION

When $t = 0$, the integral simplifies to ω_1/π. Since

$$\lim_{t \to 0} \frac{1}{\pi t} \sin(\omega_1 t) = \omega_1/\pi$$

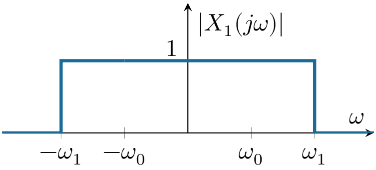
Therefore, we usually write

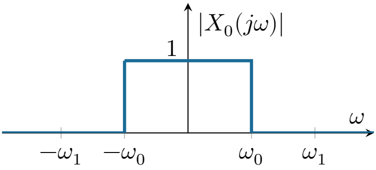
$$\frac{1}{\pi t} \sin(\omega_1 t) \quad \text{or} \quad \frac{\omega_1}{\pi} \operatorname{sinc}\left(\frac{\omega_1 t}{\pi}\right).$$

Where $\operatorname{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$ is the normalised sinc function.

EXPLANATION

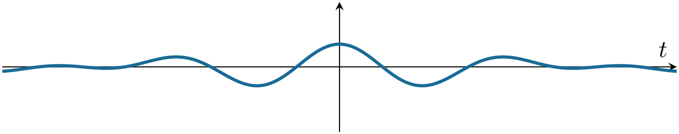
$|X(j\omega)|$ can be expressed as the difference:





Thus:

$$\frac{1}{\pi t}\sin(\omega_1 t) - \frac{1}{\pi t}\sin(\omega_0 t)$$



QUESTION 5:

(1) The Fourier transform of $x(t-2)$ is...

(2) The Fourier transform of $x(\frac{t}{2})$ is...

(i) ...$X(j\omega)e^{-j2\omega}$
(ii) ...$X(j2\omega)$
(iii) ...$2X(j2\omega)$
(iv) ...$X(j\omega - 2)$

If the Fourier transform of $x(t)$ is $X(j\omega)$, then match the two statements above:

A: **(1)**-(ii), **(2)**-(i)
B: **(1)**-(i), **(2)**-(iii)
C: **(1)**-(iv), **(2)**-(ii)
D: **(1)**-(i), **(2)**-(ii)

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A: **(1)**-(ii), **(2)**-(i)
B: **(1)**-(i), **(2)**-(iii)
C: **(1)**-(iv), **(2)**-(ii)
D: **(1)**-(i), **(2)**-(ii)

EXPLANATION

Time shifting property:

$$x(t–t_0) \Leftrightarrow e^{-j\omega t_0} X(j\omega)$$

$$x(t–2) \Leftrightarrow X(j\omega) e^{-j2\omega}$$

Time scaling property:

$$x(at) \Leftrightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

$$x\left(\frac{t}{2}\right) \Leftrightarrow 2X(j2\omega)$$