Imperial College London

MODULE 4 & 5 CLASS

Aidan Hogg & Patrick Naylor - Autumn Term 2020
ELEC50013: Signal and Systems
Department of Electrical and Electronic Engineering

Method:

1: Conceptual question posed - students individually come up initial answer **(5 mins)**

2: Explanation/discussion of correct answer **(5 mins)**

QUESTION 1:

A continuous-time LTI system is described by the differential equation:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

By using Fourier transforms and partial fraction expansion, find the impulse response $h(t)$?

A: $e^{-2t}u(t) + e^{-4t}u(t)$

B: $e^{-2t}u(t) - e^{-4t}u(t)$

C: $e^{4t}u(t) - e^{2t}u(t)$

D: $e^{2t}u(t) - e^{4t}u(t)$

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EXPLANATION

Differential equation:

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

Taking the Fourier transform of both sides:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{-\omega^2 + 6j\omega + 8}$$

Using partial fractions:

$$H(j\omega) = \frac{1}{j\omega + 2} - \frac{1}{j\omega + 4}$$

Taking the inverse Fourier transform:

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

QUESTION 2:

Consider a system with impulse response:

$$h(t) = e^{-t/2}u(t) + e^{-t}u(t) + e^{t}u(-t)$$

Where are the poles located and what is the ROC?

A: Poles at $s = \{-1, 0\},\ \{0.5, 0\},\ \{1, 0\}$,
ROC: $-1 < \Re(s) < 0.5$

B: Poles at $s = \{-0.5, 0\},\ \{-1, 0\},\ \{1, 0\}$,
ROC: $-1 < \Re(s) < 1$

C: Poles at $s = \{-0.5, 0\},\ \{-1, 0\},\ \{1, 0\}$,
ROC: $-0.5 < \Re(s) < 1$

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C: Poles at $s = \{-0.5, 0\},\ \{-1, 0\},\ \{1, 0\}$,
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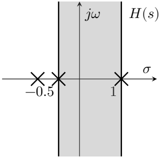
EXPLANATION

$$h(t) = e^{-t/2}u(t) + e^{-t}u(t) + e^{t}u(-t)$$



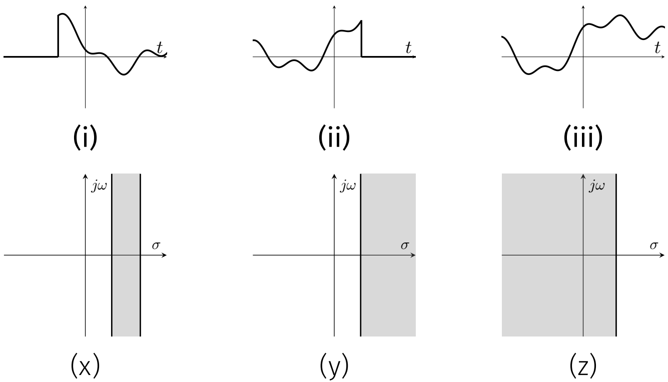
$$\begin{aligned} H(s) &= \int_{-\infty}^{\infty} x(t)e^{-st}dt \\ &= \int_{0}^{\infty} e^{-(s+0.5)t}dt + \int_{0}^{\infty} e^{-(s+1)t}dt + \int_{-\infty}^{0} e^{-(s-1)t}dt \\ &= \left[\frac{-1}{s+0.5}e^{-(s+0.5)t}\right]_0^{\infty} + \left[\frac{-1}{s+1}e^{-(s+1)t}\right]_0^{\infty} + \left[\frac{-1}{s-1}e^{-(s-1)t}\right]_{-\infty}^{0} \\ &= \frac{1}{s+0.5} + \frac{1}{s+1} - \frac{1}{s-1} \end{aligned}$$

ROC: $\Re(s) > -0.5$ and $\Re(s) > -1$ and $\Re(s) < 1$

ROC: $-0.5 < \Re(s) < 1$

Therefore, poles at $s = \{-1, 0\}$, $s = \{-0.5, 0\}$ and $s = \{1, 0\}$

QUESTION 3:

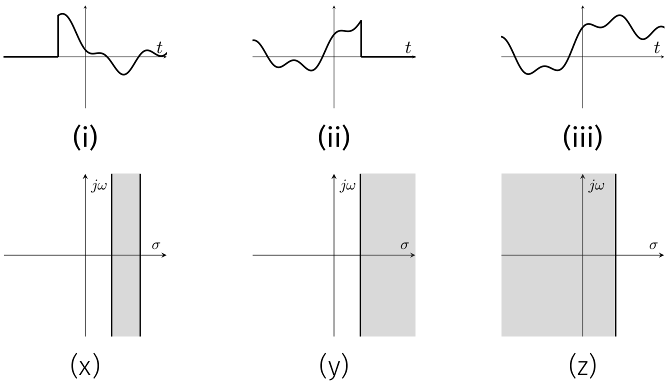


What is the correct mapping between these signals and their ROCs?

A: **(i)** : (x), **(ii)** : (y), **(iii)** : (z)

B: **(i)** : (z), **(ii)** : (y), **(iii)** : (x)

C: **(i)** : (y), **(ii)** : (z), **(iii)** : (x)



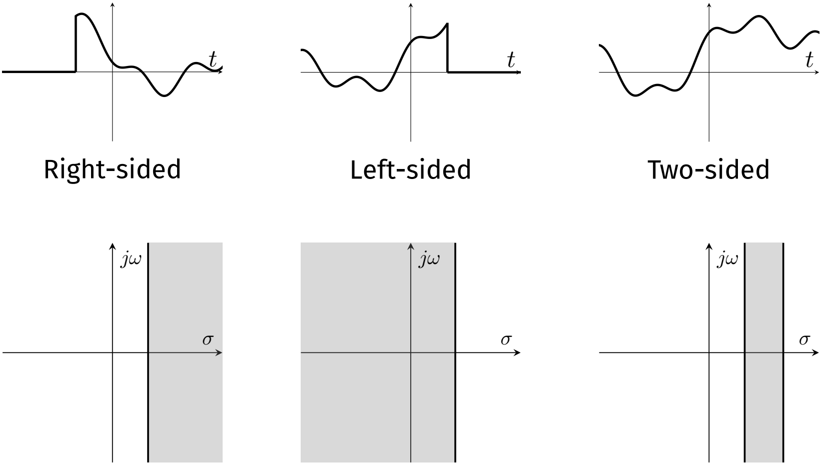
What is the correct mapping between these signals and their ROCs?

A: **(i)** : (x), **(ii)** : (y), **(iii)** : (z)

B: **(i)** : (z), **(ii)** : (y), **(iii)** : (x)

C: **(i)** : (y), **(ii)** : (z), **(iii)** : (x)

EXPLANATION



For which of these signals does the Fourier transform exist?

QUESTION 4:

An absolutely integrable signal $x(t)$, i.e. its Fourier transform $X(j\omega)$ exists, is known to have a pole at $s = 2$.

Which of these statements are true? (multiple options allowed)

A: $x(t)$ could be of finite duration

B: $x(t)$ could be left sided

C: $x(t)$ could be right sided

D: $x(t)$ could be two sided

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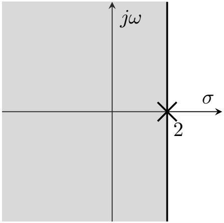
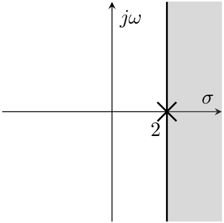
EXPLANATION

If $x(t)$ is absolutely integrable and of finite duration, then the ROC is the entire s-plane (the Laplace transform integral is finite, i.e. $X(s)$ exists, for any s).

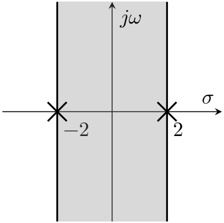
A signal $x(t)$ is absolutely integrable, i.e. its Fourier transform $X(j\omega)$ exists, if and only if the ROC of the corresponding Laplace transform $X(s)$ contains the imaginary axis $\Re(s) = 0$ or $s = j\omega$.



Left-sided



Right-sided

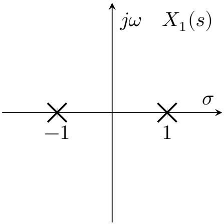


Two-sided

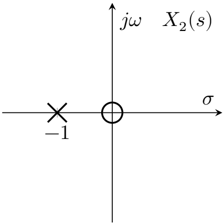
QUESTION 5:

Determine which of the following pole-zero diagrams could represent Laplace transforms of even functions of time.

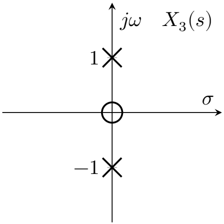
(multiple options allowed)



A



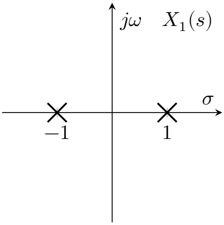
B



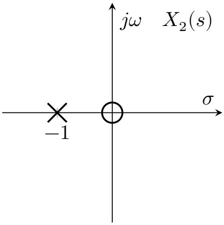
C

Determine which of the following pole-zero diagrams could represent Laplace transforms of even functions of time.

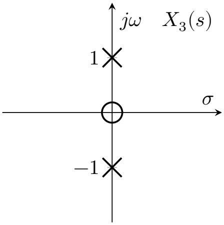
(multiple options allowed)



A



B



C

EXPLANATION

If $x(t)$ is an even function of time, then the corresponding Laplace transform must be symmetric about the point $s = 0$.

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} = \int_{-\infty}^{\infty} x(-t)e^{-st} = \int_{-\infty}^{\infty} x(t)e^{st} = X(-s)$$

Thus the pole-zero diagram must also be symmetric about $s = 0$.

This means that $X_2(s)$ cannot correspond to an even function of time.

EXPLANATION

For $X(s)$ to be symmetric about $s = 0$, then **the region of convergence must be symmetric** about the $j\omega$ axis. Symmetry is not possible for $X_3(s)$, which must be either right-sided or left-sided but not both.

$X_1(s)$ has partial fraction expansions:

$$\frac{A}{s+1} + \frac{B}{s-1}$$

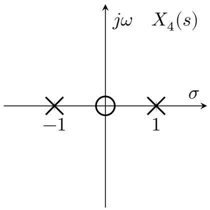
To get no finite zeros as in $X_1(s)$, $A = -B$. The corresponding time function is:

$$x_1(t) = A\big(e^{-t}u(t) + e^{t}u(-t)\big)$$

which is an even function of time for all values of A. Thus, only $x_1(t)$ is an even function of t.

EXPLANATION

Note that symmetry of the pole-zero diagram is **necessary but not sufficient** for symmetry of the corresponding time function, for example $x_4(t)$.



In this case $x_4(t)$ has a similar partial fraction expansion to $x_1(t)$:

$$\frac{A}{s+1} + \frac{B}{s-1}$$

But to get a zero at $s = 0$, $A = B$. The corresponding time function is:

$$x_4(t) = A\big(e^{-t}u(t) - e^{t}u(-t)\big)$$

which is an odd function of time for all values of A.