Imperial College London

MODULE 2 CLASS

Aidan Hogg & Patrick Naylor - Autumn Term 2020
ELEC50013: Signal and Systems
Department of Electrical and Electronic Engineering

Why ‘Peer Instruction’?

1: It forces students to engage in the class

2: Students in the past have given very positive feedback

3: In the exam most students do well in the mathematical questions but clearly have very limited conceptual understanding of the questions they are solving **(plug ’n chug)**

But what if I am struggling with a particular problem?

Piazza: This is where you can ask detailed questions on problems you are struggling with.

Sign up on Blackboard!

Method:

1: Conceptual question posed - students individually come up initial answer **(5 mins)**

2: Explanation/discussion of correct answer **(5 mins)**

QUESTION 1:

The normal arithmetic rules for multiplication are:

Identity: $x[n] * \delta[n] = x[n]$ **or** $x(t) * \delta(t) = x(t)$

Commutative: $x[n] * v[n] = v[n] * x[n]$ **or** $x(t) * v(t) = v(t) * x(t)$

Associative: $x[n] * (v[n] * w[n]) = (x[n] * v[n]) * w[n]$ **or**

$$x(t) * (v(t) * w(t)) = (x(t) * v(t)) * w(t)$$

Distributive: $x[n] * (v[n] + w[n]) = x[n] * v[n] + x[n] * w[n]$ **or**

$$x(t) * (v(t) + w(t)) = x(t) * v(t) + x(t) * w(t)$$

What normal arithmetic rules does convolution obey? (multiple options allowed)

A: Identity

B: Commutative

C: Associative

D: Distributive

The normal arithmetic rules for multiplication are:

Identity: $x[n] * \delta[n] = x[n]$ **or** $x(t) * \delta(t) = x(t)$

Commutative: $x[n] * v[n] = v[n] * x[n]$ **or** $x(t) * v(t) = v(t) * x(t)$

Associative: $x[n] * (v[n] * w[n]) = (x[n] * v[n]) * w[n]$ **or**

$x(t) * (v(t) * w(t)) = (x(t) * v(t)) * w(t)$

Distributive: $x[n] * (v[n] + w[n]) = x[n] * v[n] + x[n] * w[n]$ **or**

$x(t) * (v(t) + w(t)) = x(t) * v(t) + x(t) * w(t)$

What normal arithmetic rules does convolution obey? (multiple options allowed)

A: Identity

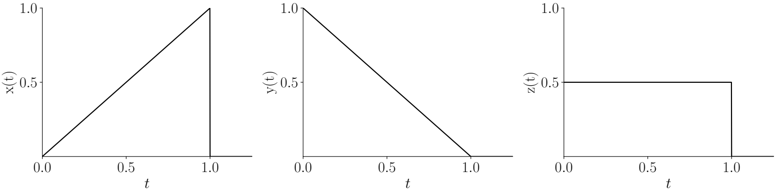
B: Commutative

C: Associative

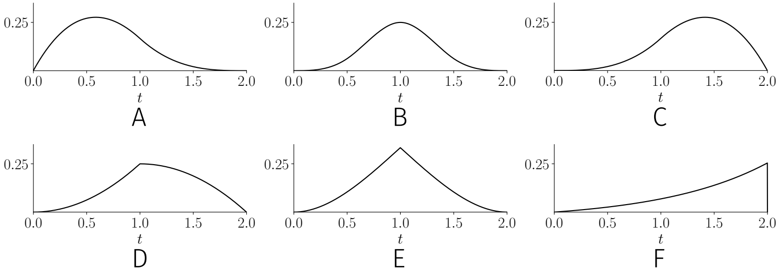
D: Distributive

QUESTION 2:

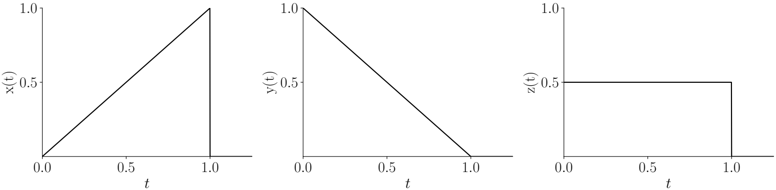
Consider the convolution of two of the following signals:



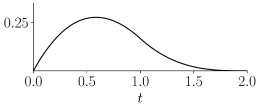
Which of the following signals can be constructed? (multiple options allowed)



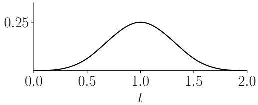
Consider the convolution of two of the following signals:



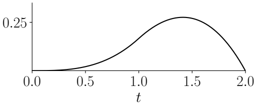
Which of the following signals can be constructed? (multiple options allowed)



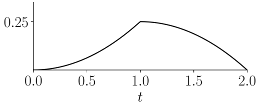
A



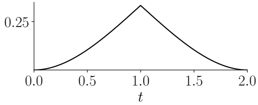
B



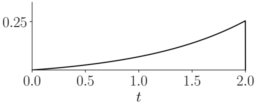
C



D



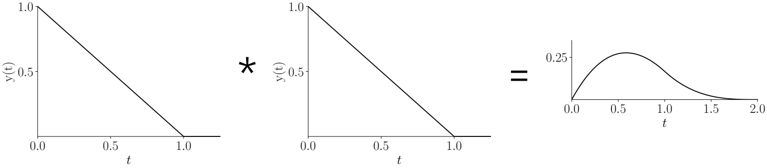
E



F

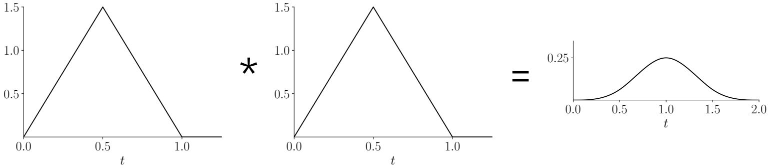
EXPLANATION

A:



Must be asymmetric with large output at early times and smaller output at later times.

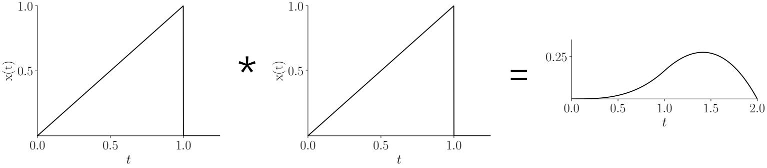
B:



The output is symmetric, which could happen if one of the inputs is a flipped-in-time version of the other input. There are only a few such options. One is $z(t) * z(t)$ but that would result in a triangle-shaped output. Another symmetric option is $x(t) * y(t)$ (or equivalently $y(t) * x(t)$) but that would result in a point at $t = 1$. **None of the provided functions could result in this output.**

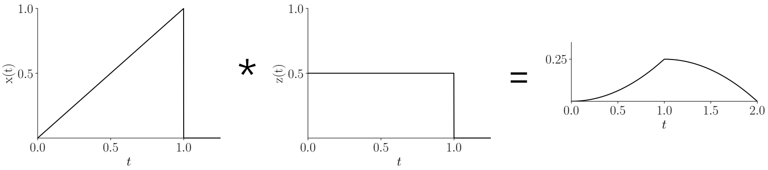
EXPLANATION

C:

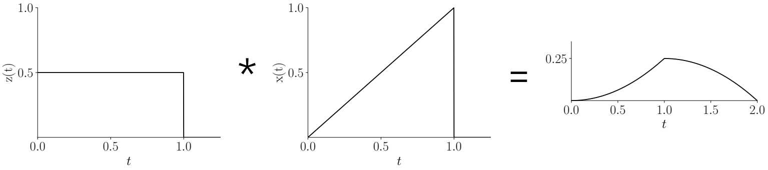


Must be asymmetric with small output at early times and larger output at later times

EXPLANATION

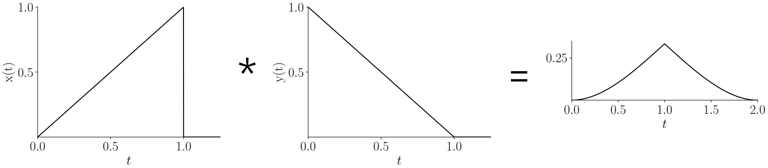
D: 

or

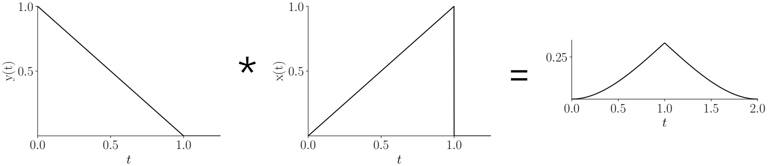
D: 

The first interval looks like the integral of $x(t)$. The second interval looks like the first interval, but shifted and flipped in time. Thus the answer is $x(t) * z(t)$ or $z(t) * z(t)$.

EXPLANATION

E: 

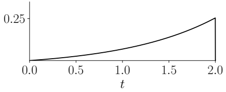
or

E: 

The output is symmetric, which could happen if one of the inputs is a flipped-in-time version of the other input. There are only a few such options. One is $z(t) * z(t)$ — but that would result in a triangle-shaped output. Another symmetric option is $x(t) * y(t)$ (or equivalently $y(t) * x(t)$) which gives this result

EXPLANATION

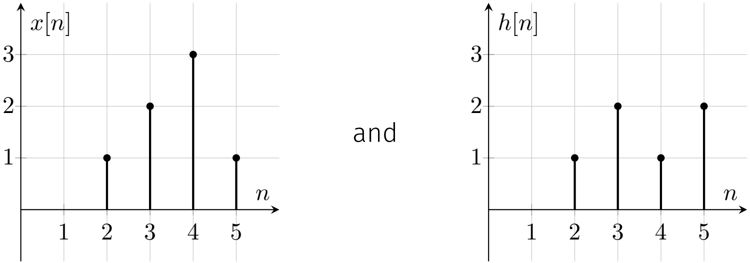
F:



The step discontinuity in this result could only result if one of the convolved functions contains an impulse. **None of the provided functions could result in this output.**

QUESTION 3:

Find the convolution of $y[n] = x[n] * h[n]$:



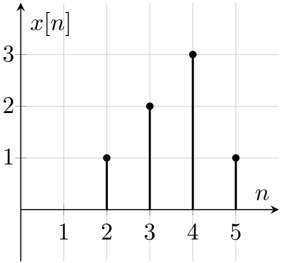
A: $y[n] = \{1, 4, 8, 11, 9, 7, 2\}$, origin at 2

B: $y[n] = \{1, 4, 8, 11, 9, 7, 2\}$, origin at 4

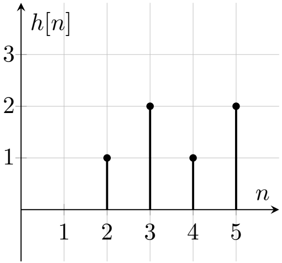
C: $y[n] = \{2, 7, 9, 11, 8, 4, 1\}$, origin at 2

D: $y[n] = \{1, 4, 8, 10, 7, 2, 1\}$, origin at 4

Find the convolution of $y[n] = x[n] * h[n]$:



and



A: $y[n] = \{1, 4, 8, 11, 9, 7, 2\}$, origin at 2

B: $y[n] = \{1, 4, 8, 11, 9, 7, 2\}$, origin at 4

C: $y[n] = \{2, 7, 9, 11, 8, 4, 1\}$, origin at 2

D: $y[n] = \{1, 4, 8, 10, 7, 2, 1\}$, origin at 4

EXPLANATION

Simple Trick Method

There is a also a very simple trick that can be used to calculate the convolution:

$$
\begin{array}{ccccccc}
 & & -2 & -1 & 0 & 1 & 2 \\
 & & & x[-1] & x[0] & x[1] & x[2] \\
 & & h[-2] & h[-1] & h[0] & h[1] & \\
\hline
 & & & h[1]x[-1] & h[1]x[0] & h[1]x[1] & h[1]x[2] \\
 & & h[0]x[-1] & h[0]x[0] & h[0]x[1] & h[0]x[2] & \times \\
 & h[-1]x[-1] & h[-1]x[0] & h[-1]x[1] & h[-1]x[2] & \times & \\
h[-2]x[-1] & h[-2]x[0] & h[-2]x[1] & h[-2]x[2] & \times & & \\
\hline
y[-3] & y[-2] & y[-1] & y[0] & y[1] & y[2] & y[3]
\end{array}
$$

EXPLANATION

Simple Trick Method

This appoach makes this question easy.

$x[n]$			1	2	3	1
$h[n]$			1	2	1	2
			2	4	6	2
		1	2	3	1	$\times$
	2	4	6	2	$\times$	
1	2	3	1	$\times$		
1	4	8	11	9	7	2
$y[4]$	$y[5]$	$y[6]$	$y[7]$	$y[8]$	$y[9]$	$y[10]$

QUESTION 4:

If the response of LTI continuous time system to unit step input is:

$$s(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

What is the impulse response of the system?

A: $h(t) = \frac{1}{2}t + \frac{1}{4}e^{-2t}$

B: $h(t) = e^{-2t}$

C: $h(t) = (1 - e^{-2t})$

D: $h(t) = -\frac{1}{2}e^{-2t}$

If the response of LTI continuous time system to unit step input is:

$$s(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

What is the impulse response of the system?

A: $h(t) = \frac{1}{2}t + \frac{1}{4}e^{-2t}$

B: $h(t) = e^{-2t}$

C: $h(t) = (1 - e^{-2t})$

D: $h(t) = -\frac{1}{2}e^{-2t}$

EXPLANATION

Given a system with impulse response $h(t)$ the step response is:

$$s(t) = \int_{-\infty}^{t} h(\tau) d\tau$$

We can invert this relationship to express the impulse response in term of the step response:

$$h(t) = \frac{d}{dt} s(t)$$

Thus the impulse response of this system is:

$$h(t) = \frac{d}{dt}\left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right) = e^{-2t}$$

What is the equivalent relationship in the discrete-time domain?

QUESTION 5:

Which of the following statements are true? (multiple options allowed)

A: The forced response depends on both the input and the roots of the characteristic equation

B: If we want a system to be causal and linear time-invariant then the initial conditions must be zero (‘initial rest’)

C: The roots of the system’s characteristic equation directly relate to the stability characteristics of an LTI system

Which of the following statements are true? (multiple options allowed)

A: The forced response depends on both the input and the roots of the characteristic equation

B: If we want a system to be causal and linear time-invariant then the initial conditions must be zero ('initial rest')

C: The roots of the system's characteristic equation directly relate to the stability characteristics of an LTI system

Natural response:

The systems output for a zero input and thus describes any stored energy or memory of the past represented by non-zero initial conditions.

The natural response, therefore, only depends on the characteristic equation.

Forced response:

The systems output due to the signal input assuming zero initial conditions.

The forced response, therefore, depends on both the input and the roots of the characteristic equation.

Casual linear time-invariant systems:

If we want a system to be causal and linear time-invariant then the initial conditions must be zero ('initial rest')

Causal, LTI $\Leftrightarrow$ initial rest:

$$\begin{aligned} \text{if} \quad & x(t) = 0, \quad t < t_0 \\ \text{then} \quad & y(t) = 0, \quad t < t_0 \end{aligned}$$

or

$$\begin{aligned} \text{if} \quad & x[n] = 0, \quad n < n_0 \\ \text{then} \quad & y[n] = 0, \quad n < n_0 \end{aligned}$$

Stability:

The roots of the system's characteristic equation directly relate to the stability characteristics of an LTI system.

Solution form of the homogeneous equation:

$$y^{(h)}(t) = \sum_{i=1}^{N} c_i e^{r_i t}$$

or

$$y^{(h)}[n] = \sum_{i=1}^{N} c_i r_i^n$$

For continuous-time systems we require $|e^{r_i t}|$ to be bounded so $\Re\{r_i\} < 0$.

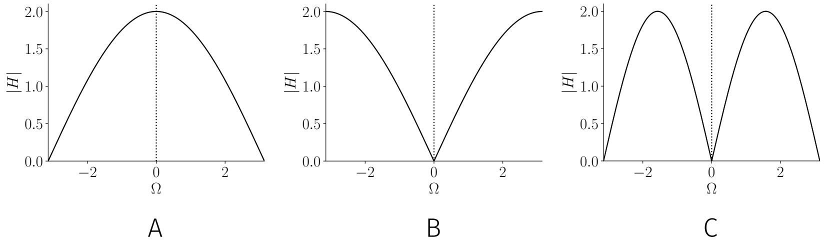
For discrete-time systems we require $|r_i^n|$ to be bounded so $|r_i| < 1 \forall i$.

QUESTION 6:

Consider the difference equation:

$$y[n] = x[n] - x[n-1]$$

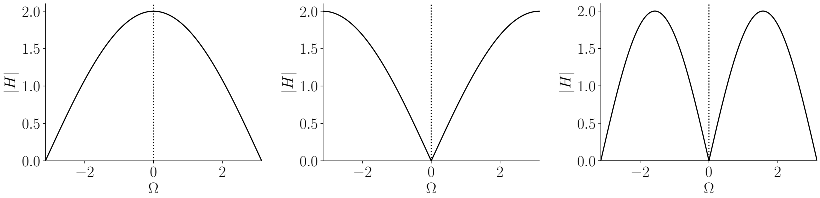
What is the frequency response?



Consider the difference equation:

$$y[n] = x[n] - x[n-1]$$

What is the frequency response?



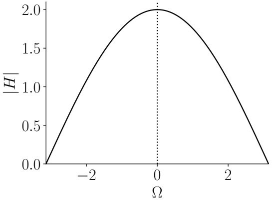
A B C

EXPLANATION

A: $h_1[n] = \{1, 1\}$

$$y[n] = x[n] * h_1[n] = x[n] + x[n-1]$$

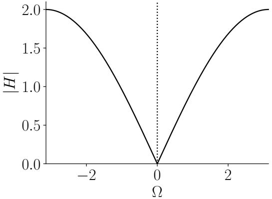
(Low-pass filter: averages)



B: $h_2[n] = \{1, -1\}$

$$y[n] = x[n] * h_2[n] = x[n] - x[n-1]$$

(High-pass filter)



C: $h_3[n] = h_1[n] * h_2[n] = \{1, 0, -1\}$

$$y[n] = x[n] * h_3[n] = x[n] - x[n-2]$$

(Band-pass filter: low-pass followed by high-Pass)