Imperial College London

MODULE 1 CLASS

Aidan Hogg & Patrick Naylor - Autumn Term 2020
ELEC50013: Signal and Systems
Department of Electrical and Electronic Engineering

PEER INSTRUCTION

Why ‘Peer Instruction’?

1: It forces students to engage in the class

2: Students in the past have given very positive feedback

3: In the exam most students do well in the mathematical questions but clearly have very limited conceptual understanding of the questions they are solving **(plug ’n chug)**

But what if I am struggling with a particular problem?

Piazza: This is where you can ask detailed questions on problems you are struggling with.

Sign up on Blackboard!

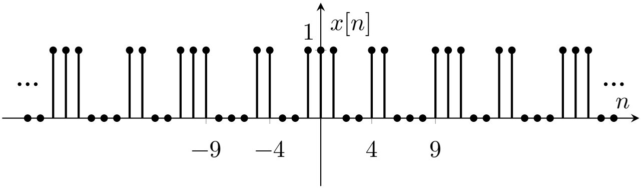
Method:

1: Conceptual question posed - students individually come up initial answer **(5 mins)**

2: Explanation/discussion of correct answer **(5 mins)**

QUESTION 1:

Consider the following 3 systems:



$$y(t) = \sin(t)\cos(t)$$

$$v[n] = (-1)^n$$

What fact is true about all these systems?

A: They are all even but not all periodic

B: They are all periodic but not all even

C: They all even and periodic

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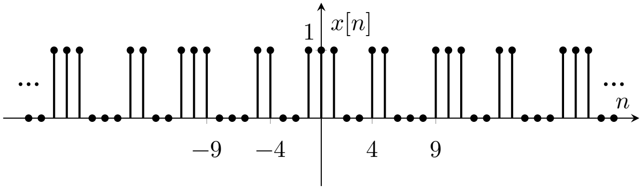

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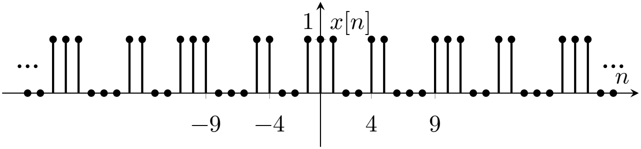
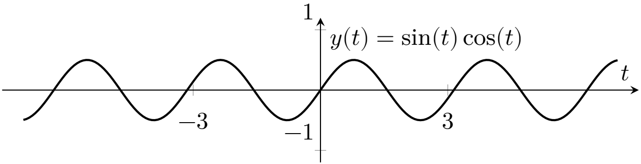
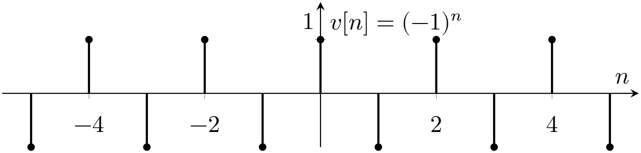
EXPLANATION



Even and Periodic



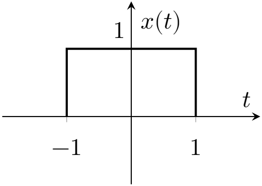
Odd and Periodic



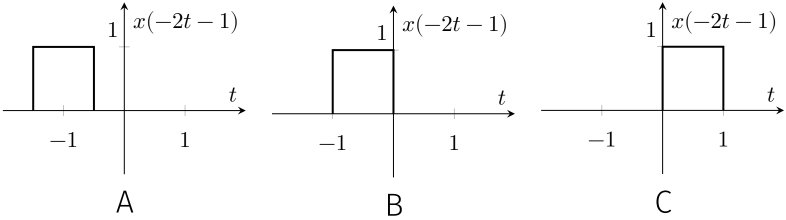
Even and Periodic

QUESTION 2:

Given $x(t)$ is:



Which of the following correctly depicts the signal: $x(-2t-1)$



ANSWER

[RELATED LECTURE EXERCISES: 4]

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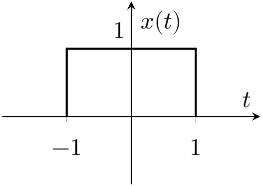

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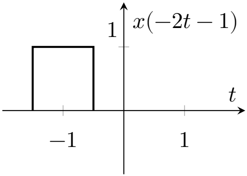
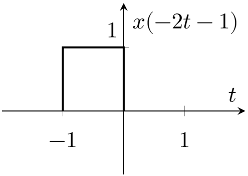
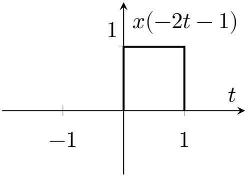


A



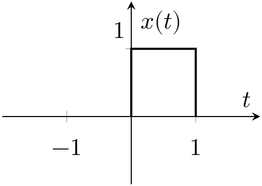
B



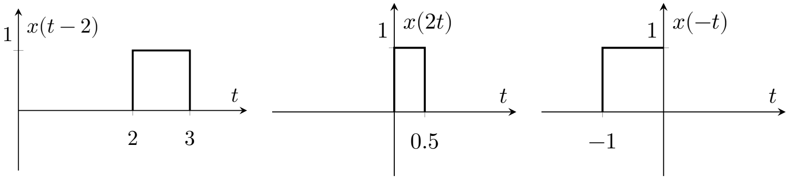
C

EXPLANATION

Given $x(t)$ is:



Basic operations on signals:

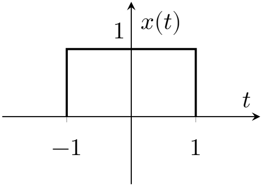


Time-shifting

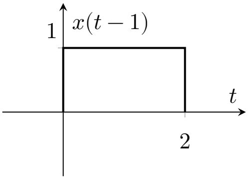
TIme-scaling

Reflection

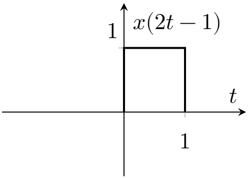
Precedence rule: Time-shift operation is always performed first and then the time scaling (and reflection)



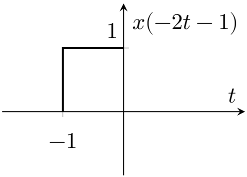
Correct order of operations:



1. Time-shifting



2. Time-scaling



3. Reflection

QUESTION 3:

When referring to a “BIBO-stable” system what is meant by the term “bounded”?

A sequence $x[n]$ is bounded iff $\exists B < \infty$ such that

A: $\sum |x[n]| < B$

B: $|x[n]| < B \; \forall \; n$

C: $\sum |x[n]|^2 < B$

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When referring to a "BIBO-stable" system what is meant by the term "bounded"?

A sequence $x[n]$ is bounded iff $\exists B < \infty$ such that

A: $\sum |x[n]| < B$ (Absolutely summable)

B: $|x[n]| < B \ \forall \ n$ (Bounded)

C: $\sum |x[n]|^2 < B$ (Finite Energy)

QUESTION 4:

Consider the following systems:

$$y(t) = \frac{d}{dt}x(t)$$

$$v[n] = 0.5x[n] + 0.5v[n-1]$$

$$w[n] = \sum_{k=-\infty}^{n} x[k+2]$$

What fact is true about all these systems?

A: They are all causal

B: They are all stable

C: They are all stable and causal

D: None of them

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D: None of them

EXPLANATION

True facts:

$$y(t) = \frac{d}{dt}x(t)$$ Stable and Causal

$$v[n] = 0.5x[n] + 0.5v[n-1]$$ Stable and Causal

$$w[n] = \sum_{k=-\infty}^{n} x[k+2]$$

$$w[n] = \sum_{k=-\infty}^{n-1} x[k+2] + x[n+2]$$ Not Stable and Non-causal

$$w[n] = w[n-1] + x[n+2]$$

QUESTION 5:

Which of these system are linear? (multiple options allowed)

$$y[n] = \sqrt{x[n]}$$

$$v(t) = \cos(x(t))$$

$$w[n] = x[2(n-1)]$$

Which of these system are linear? (multiple options allowed)

$$y[n] = \sqrt{x[n]}$$

$$v(t) = \cos(x(t))$$

$$w[n] = x[2(n-1)]$$

$y[n] = \sqrt{x[n]}$ is **NOT linear** because if

$$y_1[n] = \sqrt{x_1[n]},$$

and

$$y_2[n] = \sqrt{x_2[n]},$$

and $x[n] = \alpha x_1[n] + \beta x_2[n]$, then the output $y[n]$ corresponding to the input $x[n]$ is

$$\begin{aligned} y[n] = \sqrt{x[n]} &= \sqrt{\alpha x_1[n] + \beta x_2[n]]} \\ &\neq \alpha y_1[n] + \beta y_2[n] = \alpha\sqrt{x_1[n]} + \beta\sqrt{x_2[n]} \end{aligned}$$

$v(t) = \cos(x(t))$ is **NOT linear** because if

$$v_1(t) = \cos(x_1(t)),$$

and

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and $x(t) = \alpha x_1(t) + \beta x_2(t)$, then the output $v(t)$ corresponding to the input $x(t)$ is

$$\begin{aligned} v(t) &= \cos(\alpha x_1(t) + \beta x_2(t)) \\ &= \cos(\alpha x_1(t)) \cos(\beta x_2(t)) - \sin(\alpha x_1(t)) \sin(\beta x_2(t)) \\ &\neq \alpha \cos(x_1(t)) + \beta \cos(x_2(t)) \end{aligned}$$

EXPLANATION

$w[n] = x[2(n-1)]$ is **linear** because if

$$w_1[n] = x_1[2(n-1)],$$

and

$$w_2[n] = x_2[2(n-1)],$$

and $x[n] = \alpha x_1[n] + \beta x_2[n]$, then the output $w[n]$ corresponding to the input $x[n]$ is

$$\begin{aligned} w[n] &= x[2(n-1)] = \alpha x_1[2(n-1)] + \beta x_2[2(n-1)] \\ &= \alpha w_1[n] + \beta w_2[n] \end{aligned}$$

QUESTION 6:

Which of these statements are true? (multiple options allowed)

A: It is possible for a noncausal system to possess memory

B: It is possible for a signal to be neither a power or energy signal

C: It is possible for a signal to be linear but not time invariant

D: None of them

Which of these statements are true? (multiple options allowed)

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Memory

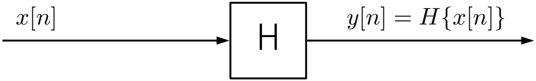
A system is seen to possess memory if the output signal depends on values of the input signal at any past (or future) time

Causality

There are a small number of sequences that are worth taking note of:

Right-sided:	$h[n] = 0$ for $n < N_{\text{min}}$
Left-sided:	$h[n] = 0$ for $n > N_{\text{max}}$
Finite length:	$h[n] = 0$ for $n \notin [N_{\text{min}}, N_{\text{max}}]$
Causal:	$h[n] = 0$ for $n < 0$
Anticausal:	$h[n] = 0$ for $n > 0$

Linearity and time-invariance



A linear time-invariant system can be defined by two properties:

Linear: $H\{\alpha u[n] + \beta v[n]\} = \alpha H\{u[n]\} + \beta H\{v[n]\}$

Time Invariant: $y[n] = H\{x[n]\} \Rightarrow y[n-r] = H\{x[n-r]\} \forall r$

Note: The behaviour of an LTI system is completely defined by its impulse response: $h[n] = H\{\delta[n]\}$

Power and energy signals

- A signal is called a power signal if $0 \leq P_\infty \leq \infty$
- A signal is called an energy signal if $E_\infty \leq \infty$
- A signal can be either a power signal an energy signal or nether type.
- A signal cannot be both an energy signal and a power signal

An signal that is nether a power or energy signal would have infinite power (and also infinite energy).

For example an exponential signal:

$$x(t) = e^t$$