Imperial College London

REVISION CLASS 1

Aidan Hogg - 21 November 2019
ELEC96010 (EE3-07): Digital Signal Processing
Department of Electrical and Electronic Engineering

Method:

1: Conceptual question posed - students think quietly on their own and report initial answers on Mentimeter **(3 mins)**

2: Students discuss their answers in small groups **(2 mins)**

3: Explanation/discussion of correct answer **(3 mins)**

Consider the following statements:

1: Allpass filters have mirror image numerator and denominator coefficients

2: In an allpass filter, the zeros are the poles reflected in the unit circle

3: Allpass filters have a gain magnitude of 1 even with coefficient errors

Which of these statements are true?

A: 1

B: 1 and 2

C: All 3 of them

ANSWER

Consider the following statements:

1: Allpass filters have mirror image numerator and denominator coefficients

2: In an allpass filter, the zeros are the poles reflected in the unit circle

3: Allpass filters have a gain magnitude of 1 even with coefficient errors

Which of these statements are true?

A: 1

B: 1 and 2

C: All 3 of them

Allpass filters have mirror image numerator and denominator coefficients

$$b[n] = a[N-n] \quad \Leftrightarrow \quad B(z) = z^{-N}A(z^{-1})$$

$$H(e^j\omega) = \frac{\sum_{r=0}^{M} b[r]e^{-j\omega r}}{\sum_{r=0}^{M} b[r]e^{-j\omega(M-r)}} = e^{j\omega M}\frac{\sum_{r=0}^{M} b[r]e^{-j\omega r}}{\sum_{r=0}^{M} b[r]e^{j\omega r}}$$

$$\Rightarrow \quad |H(e^{j\omega})| \equiv 1 \forall \omega$$

Allpass filters have a gain magnitude of 1 even with coefficient errors

First Order:

$$H(z) = \frac{-p + z^{-1}}{1 - pz^{-1}} = -p\frac{1 - p^{-1}z^{-1}}{1 - pz^{-1}}$$

Poles at p and zero at p^{-1}: 'reflected in unit circle'

Constant distance ratio $\left|e^{j\omega} - p\right| = |p|\left|e^{j\omega} - \frac{1}{p}\right| \forall\omega$

Consider the following system:

$$y[n] = 2x[n] - 3x[n-1] + x[n-2]$$

Where are the poles and zeros located?

A: Zeros at $z = \{2, 0\}$ and $\{1, 0\}$ Poles at $z = \{0, 0\} \times 2$

B: Zeros at $z = \{2, 0\}$ and $\{1, 0\}$ Pole at $z = \{0, 0\}$

C: Zeros at $z = \{1/2, 0\}$ and $\{1, 0\}$ Poles at $z = \{0, 0\} \times 2$

Consider the following system:

$$y[n] = 2x[n] - 3x[n-1] + x[n-2]$$

Where are the poles and zeros located?

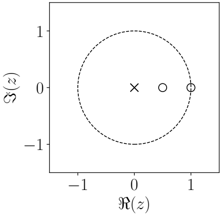
A: Zeros at $z = \{2, 0\}$ and $\{1, 0\}$ Poles at $z = \{0, 0\} \times 2$

B: Zeros at $z = \{2, 0\}$ and $\{1, 0\}$ Pole at $z = \{0, 0\}$

C: **Zeros at $z = \{1/2, 0\}$ and $\{1, 0\}$ Poles at $z = \{0, 0\} \times 2$**

Solution

$$y[n] = 2x[n] - 3x[n-1] + x[n-2]$$
$$Y(z) = 2X(z) - 3z^{-1}X(z) + z^{-2}X(z)$$
$$Y(z) = [z^{-2} - 3z^{-1} + 2]X(z)$$
$$Y(z) = [(z^{-1} - 2)(z^{-1} - 1)]X(z)$$



Therefore

$$\text{Zeros at } z = \left\{\frac{1}{2}, 0\right\} \text{ and } \left\{1, 0\right\}$$

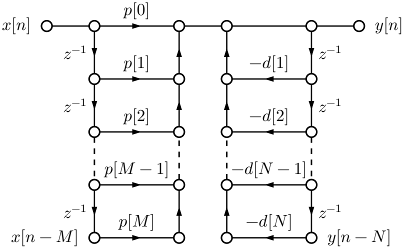
$$\text{Poles at } z = \left\{0, 0\right\} \text{ and } \left\{0, 0\right\}$$

GO TO WWW.MENTI.COM AND USE THE CODE 70 44 50

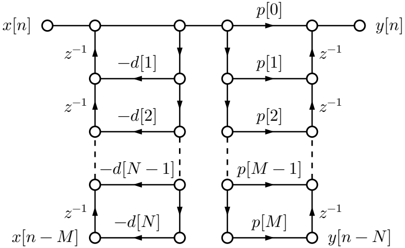
Consider the filter:

$$H(z) = \frac{p[0] + p[1]z^{-1} + \cdots + p[M]z^{-M}}{1 + d[1]z^{-1} + \cdots + d[N]z^{-N}}$$

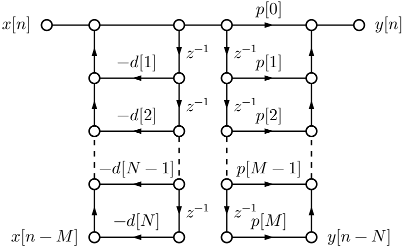
Which digital filter structure is implementing Direct Form I Transposed?



A



B



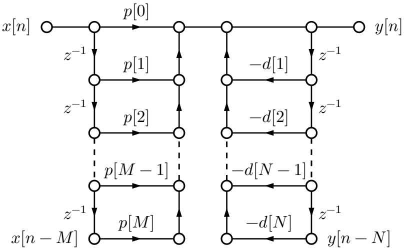
C

ANSWER

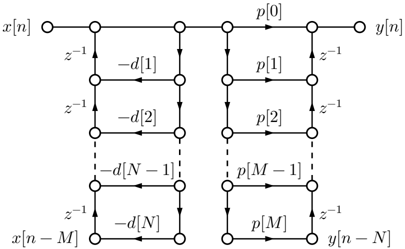
Consider the filter:

$$H(z) = \frac{p[0] + p[1]z^{-1} + \cdots + p[M]z^{-M}}{1 + d[1]z^{-1} + \cdots + d[N]z^{-N}}$$

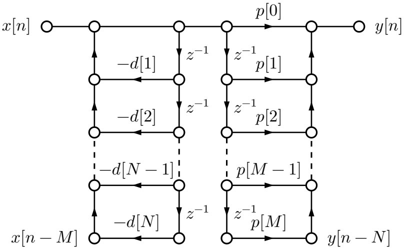
Which digital filter structure is implementing Direct Form I Transposed?



A



B

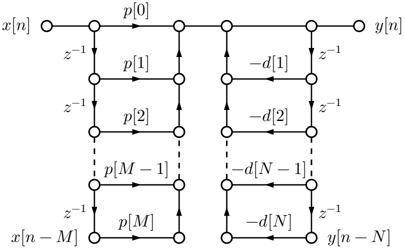


C

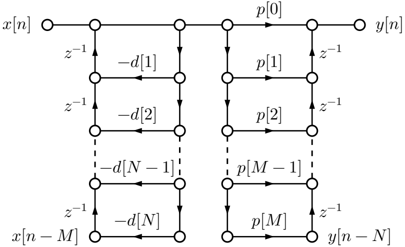
Consider the filter:

$$H(z) = \frac{p[0] + p[1]z^{-1} + \cdots + p[M]z^{-M}}{1 + d[1]z^{-1} + \cdots + d[N]z^{-N}}$$

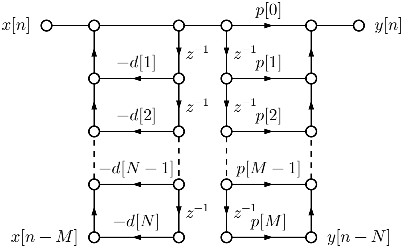
Which digital filter structure is implementing Direct Form I Transposed?



Direct Form I



Direct Form I Transposed



Direct Form II

DIRECT FORM I

If an IIR filter has a transfer function:

$$H(z) = \frac{P(z)}{D(z)} = \frac{p[0] + p[1]z^{-1} + p[2]z^{-2} + \cdots + p[M-1]z^{-(M-1)} + p[M]z^{-M}}{1 + d[1]z^{-1} + d[2]z^{-2} + \cdots + d[N-1]z^{-(N-1)} + d[N]z^{-N}}$$

Then direct forms use coefficients $d[k]$ and $p[k]$ directly. This can be implemented as a cascade of two filter sections where:

$$H_1(z) = \frac{W(z)}{X(z)} = P(z) = p[0] + p[1]z^{-1} + p[2]z^{-2} + \cdots + p[M-1]z^{-(M-1)} + p[M]z^{-M}$$

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{D(z)} = \frac{1}{1 + d[1]z^{-1} + d[2]z^{-2} + \cdots + d[N-1]z^{-(N-1)} + d[N]z^{-N}}$$

Note that $H_1(z)$ can be seen as an FIR filter and the time-domain representation of $H_2(z)$ is given by:

$$y[n] = w[n] - d[1]y[n-1] - d[2]y[n-2] - \cdots - d[N]y[n-N]$$

Direct form I can be viewed as $P(z)$ followed by $\frac{1}{D(z)}$.

Direct form II implements $\frac{1}{D(z)}$ followed by $P(z)$

Transposed Forms

It is also possible to convert any structure into an equivalent transposed form. This is achieved in the following way:

1: Reverse direction of each interconnection

2: Reverse direction of each multiplier

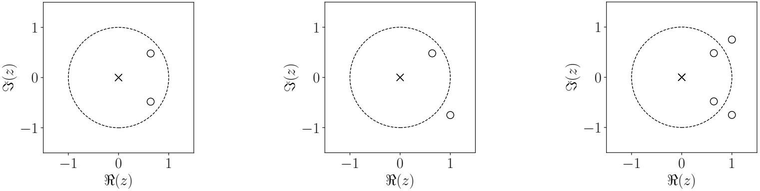
3: Change junctions to adders and vice-versa

4: Interchange the input and output signals

Check: A valid structure must never have any feedback loops that don't go through a delay (z^{-1} block).

An FIR filter $B(e^{j\omega})$ is determined by the zeros of

$$z^M B(z) = \sum_{r=0}^{M} b[M-r] z^r$$



1 2 3

Which FIR filter has symmetric coefficients $b[n]$?

A: 1 and 3

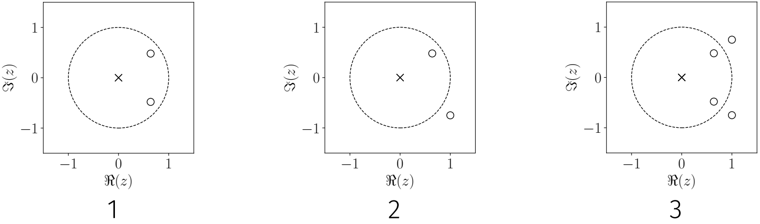
B: 2 and 3

C: 3

ANSWER

An FIR filter $B(e^{j\omega})$ is determined by the zeros of

$$z^M B(z) = \sum_{r=0}^{M} b[M-r] z^r$$



Which FIR filter has symmetric coefficients $b[n]$?

A: 1 and 3

B: 2 and 3

C: 3

Symmetric properties:

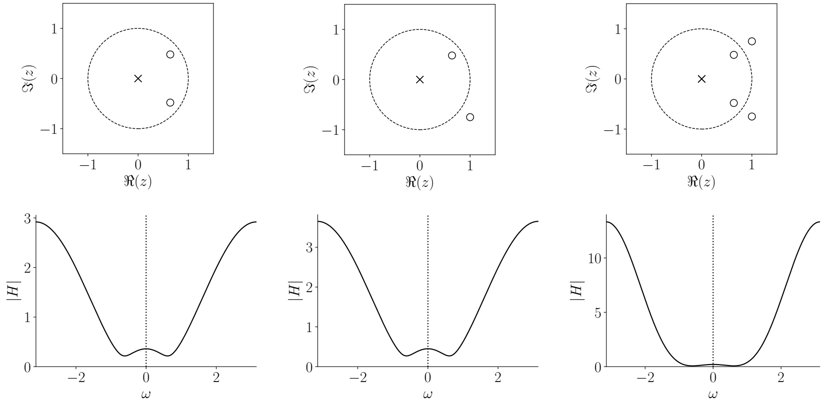
$$\text{Real } b[n] \Rightarrow \text{conjugate zero pairs: } z \Rightarrow z^*$$

$$\text{Symmetric: } b[n] = b[M-n] \Rightarrow \text{reciprocal zero pairs: } z \Rightarrow z^{-1}$$

$$\text{Real \& Symmetric } b[n] \Rightarrow \text{conjugate and reciprocal groups of four (else pairs on the real axis)}$$

EXPLANATION

Real	Symmetric	Real & Symmetric
[1, -1.28, 0.64]	[1, -1.64 + 0.27j, 1]	[1,-3.28, 4.7625, -3.28, 1]



In all of the proofs below, we assume that $z = z_0$ is a root of $B(z)$ so that $B(z_0) = \sum_{r=0}^{M} b[r] z_0^{-r} = 0$ and then we prove that this implies that other values of z also satisfy $B(z) = 0$.

(1) Real $b[n]$

$$\begin{aligned} B(z_0^*) &= \sum_{r=0}^{M} b[r](z_0^*)^{-r} \\ &= \sum_{r=0}^{M} b^*[r](z_0^*)^{-r} && \text{sine } b[r] \text{ is real} \\ &= \left(\sum_{r=0}^{M} b[r] z_0^{-r} \right)^* && \text{take complex conjugate} \\ &= 0^* = 0 && \text{since } B(z_0) = 0 \end{aligned}$$

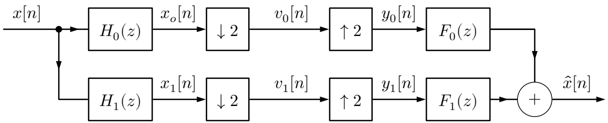
EXPLANATION

In all of the proofs below, we assume that $z = z_0$ is a root of $B(z)$ so that $B(z_0) = \sum_{r=0}^{M} b[r] z_0^{-r} = 0$ and then we prove that this implies that other values of z also satisfy $B(z) = 0$.

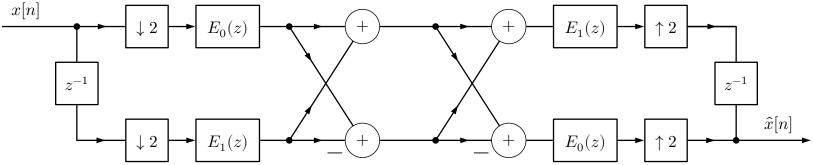
(2) Symmetric: $b[n] = b[M-n]$

$$\begin{aligned}
B(z_0^{-1}) &= \sum_{r=0}^{M} b[r] z_0^{r} \\
&= \sum_{n=0}^{M} b[M-n] z_0^{M-n} && \text{substitute } r = M-n \\
&= z_0^{M} \sum_{n=0}^{M} b[M-n] z_0^{-n} && \text{take out } z_0^{M} \text{ factor} \\
&= z_0^{M} \sum_{n=0}^{M} b[n] z_0^{-n} && \text{since } b[M-n] = b[n] \\
&= z_0^{M} \times 0 = 0 && \text{since } B(z_0) = 0
\end{aligned}$$

Consider the following Quadrature Mirror Filterbank (QMF):



Which can be implemented using polyphase QMF:



What is the computational saving achieved by the polyphase QMF?

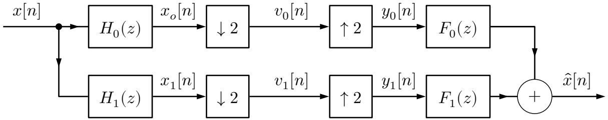
A: A factor of 2

B: A factor of 4

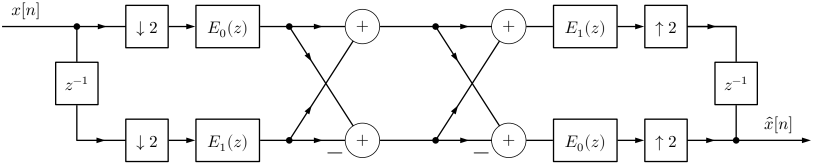
C: A factor of 8

ANSWER

Consider the following Quadrature Mirror Filterbank (QMF):



Which can be implemented using polyphase QMF:



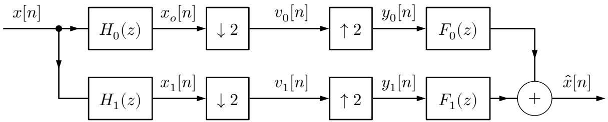
What is the computational saving achieved by the polyphase QMF?

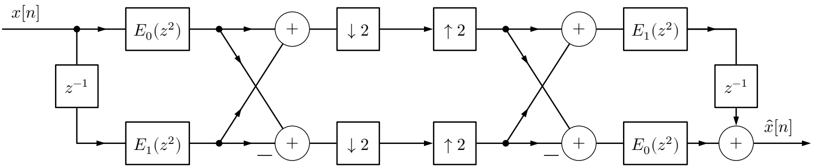
A: A factor of 2

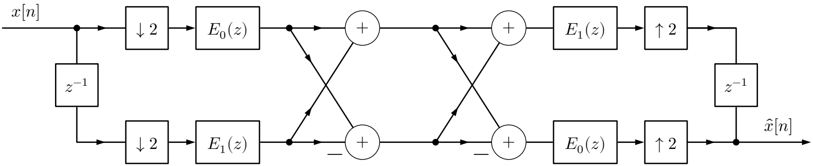
B: **A factor of 4**

C: A factor of 8

EXPLANATION

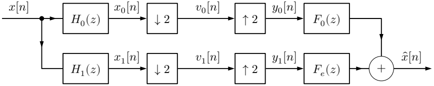


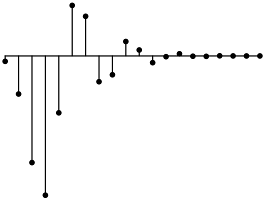




GO TO WWW.MENTI.COM AND USE THE CODE 70 44 50

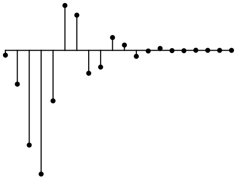
Consider the following Quadrature Mirror Filterbank (QMF):



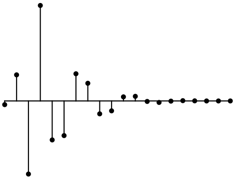


Impulse response $h_0[n]$

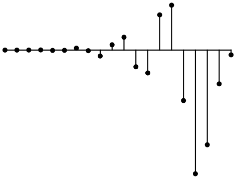
Which is the correct impulse response for $h_1[n]$?



A



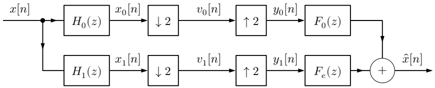
B

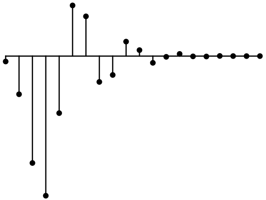


C

ANSWER

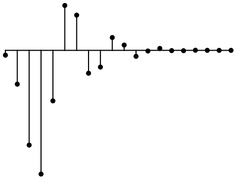
Consider the following Quadrature Mirror Filterbank (QMF):



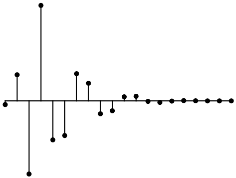


Impulse response $h_0[n]$

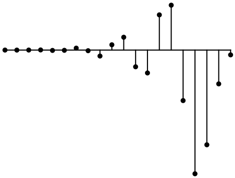
Which is the correct impulse response for $h_1[n]$?



A



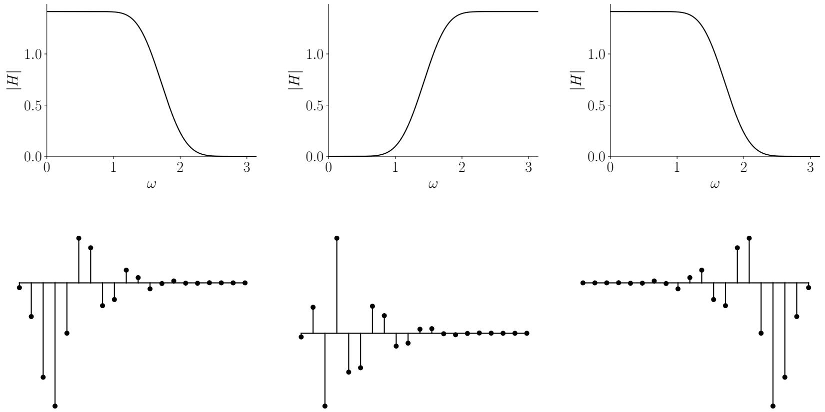
B



C

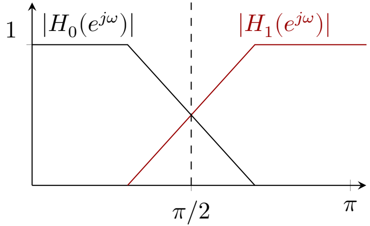
EXPLANATION

Magnitude responses:



$h_1[n] = h_0[n]$ $\quad$ $h_1[n] = (-1)^n h_0[n]$ $\quad$ $h_1[n] = h_0[N-1-n]$

Proof:



The relationship between $H_0(z)$ and $H_1(z)$ when they are quadrature mirror filters is $H_1(z) = H_0(-z)$.

The corresponding relationship between the impulse responses of these filters is $h_1[n] = (-1)^n h_0[n]$.

$$H_1(z) = H_0(-z) = \sum_{n=-\infty}^{\infty} h_0[n](-z)^{-n} = \sum_{n=-\infty}^{\infty} h_0[n](-1)^n z^{-n}$$

NEXT WEEK

There will be a class next Thursday **(Week 9 - 28/11/2019)**

Room: 403A/B

Time: 15:00 to 16:00