Imperial College London

REVISION CLASS 1

Aidan Hogg - 21 November 2019
ELEC96010 (EE3-07): Digital Signal Processing
Department of Electrical and Electronic Engineering

Method:

1: Conceptual question posed - students think quietly on their own and report initial answers on Mentimeter (3 mins)

2: Students discuss their answers in small groups (2 mins)

3: Explanation/discussion of correct answer (3 mins)

Consider the following statements:

1: Allpass filters have mirror image numerator and denominator coefficients

2: In an allpass filter, the zeros are the poles reflected in the unit circle

3: Allpass filters have a gain magnitude of 1 even with coefficient errors

Which of these statements are true?

A: 1

B: 1 and 2

C: All 3 of them

ANSWER

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Which of these statements are true?

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Allpass filters have mirror image numerator and denominator coefficients

$$[\ ] = [\ \ - \ \ ] \quad \Leftrightarrow \quad (\ ) = {}^{-} \quad (\ ^{-1})$$

$$(\quad) = \frac{\sum_{=0} [\ ]^{-}}{\sum_{=0} [\ ]^{-} \ ^{(\ -\ )}} = \frac{\sum_{=0} [\ ]^{-}}{\sum_{=0} [\ ]}$$

$$|\ (\quad)| \equiv 1$$

Allpass filters have a gain magnitude of 1 even with coefficient errors

First Order:

$$(\ ) = \frac{- \quad + \quad ^{-1}}{1 - \quad ^{-1}} = - \quad \frac{1 - \quad ^{-1} \quad ^{-1}}{1 - \quad ^{-1}}$$

Poles at and zero at $^{-1}$: ‘reflected in unit circle’

Constant distance ratio $- \quad = |\ | \quad - \frac{1}{}$

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Consider the following system:

$$[\quad] = 2\ [\quad] - 3\ [\quad - 1] + \quad [\quad - 2]$$

Where are the poles and zeros located?

A: Zeros at $\quad = \{2, 0\}$ and $\{1, 0\}$ Poles at $\quad = \{0, 0\} \times 2$

B: Zeros at $\quad = \{2, 0\}$ and $\{1, 0\}$ Pole at $\quad = \{0, 0\}$

C: Zeros at $\quad = \{1/2, 0\}$ and $\{1, 0\}$ Poles at $\quad = \{0, 0\} \times 2$

ANSWER

Consider the following system:

$$[\quad] = 2\,[\quad] - 3\,[\quad - 1] + \quad [\quad - 2]$$

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C: Zeros at $= \{1/2, 0\}$ and $\{1, 0\}$ Poles at $= \{0, 0\} \times 2$

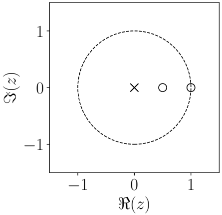
Solution

$$[\ \ ] = 2\ [\ \ ] - 3\ [\ \ - 1] + \ [\ \ - 2]$$

$$(\ \ ) = 2\ \ (\ \ ) - 3\ ^{-1}\ \ (\ \ ) + \ ^{-2}\ \ (\ \ )$$

$$(\ \ ) = [\ ^{-2} - 3\ ^{-1} + 2]\ \ (\ \ )$$

$$(\ \ ) = [(\ ^{-1} - 2)(\ ^{-1} - 1)]\ \ (\ \ )$$



Therefore

$$\text{Zeros at } \ = \{\tfrac{1}{2}, 0\} \text{ and } \{1, 0\}$$

$$\text{Poles at } \ = \{0, 0\} \text{ and } \{0, 0\}$$

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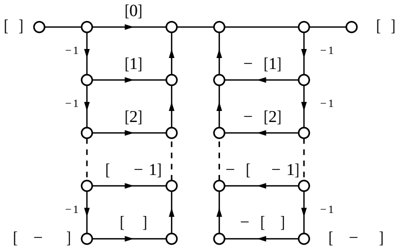
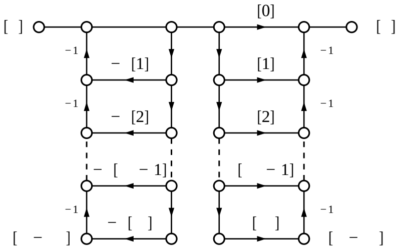
Consider the filter:

$$(\ ) = \frac{[0] + \ [1] \ ^{-1} + \quad + \ [\quad] \ ^{-}}{1 + \ [1] \ ^{-1} + \quad + \ [\quad] \ ^{-}}$$

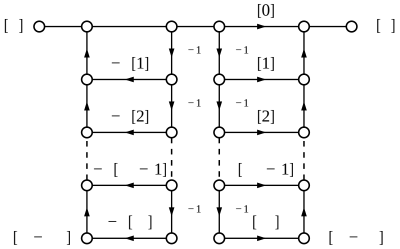
Which digital filter structure is implementing Direct Form I Transposed?



A



B



C

ANSWER

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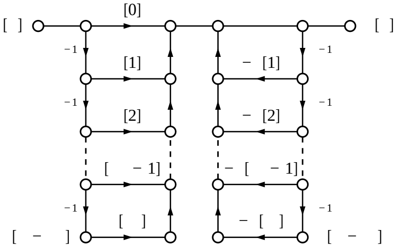
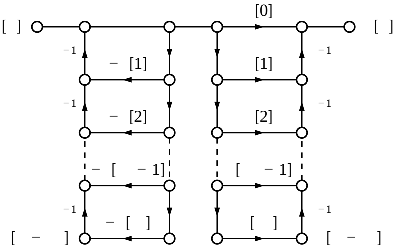
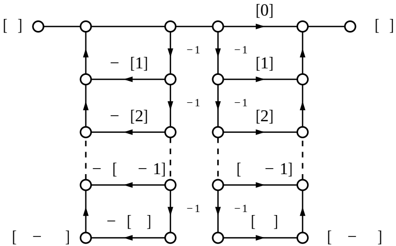


A



B



C

Consider the filter:

$$(\quad) = \frac{[0] + \quad [1] \quad ^{-1} + \quad + \quad [\quad] \quad ^{-}}{1 + \quad [1] \quad ^{-1} + \quad + \quad [\quad] \quad ^{-}}$$

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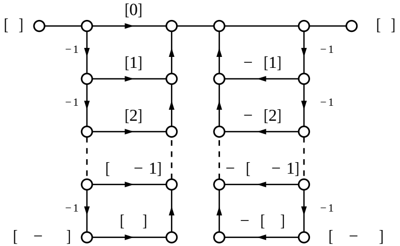
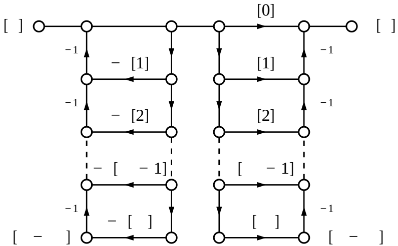
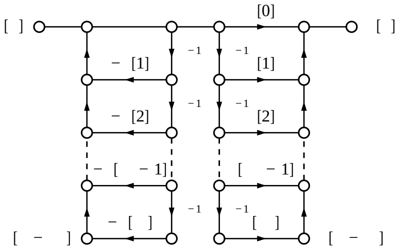


Direct Form I



Direct Form I Transposed



Direct Form II

If an IIR filter has a transfer function:

$$(\ ) = \frac{(\ )}{(\ )} = \frac{[0] + \ [1] \ ^{-1} + \ [2] \ ^{-2} + \quad + \ [\quad - 1] \ ^{-(\quad -1)} + \ [\quad] \ ^{-}}{1 + \ [1] \ ^{-1} + \ [2] \ ^{-2} + \quad + \ [\quad - 1] \ ^{-(\quad -1)} + \ [\quad] \ ^{-}}$$

Then direct forms use coefficients [ ] and [ ] directly. This can be implemented as a cascade of two filter sections where:

$$_1(\ ) = \frac{(\ )}{(\ )} = (\ ) = [0] + \ [1] \ ^{-1} + \ [2] \ ^{-2} + \quad + \ [\quad - 1] \ ^{-(\quad -1)} + \ [\quad] \ ^{-}$$

$$_2(\ ) = \frac{(\ )}{(\ )} = \frac{1}{(\ )} = \frac{1}{1 + \ [1] \ ^{-1} + \ [2] \ ^{-2} + \quad + \ [\quad - 1] \ ^{-(\quad -1)} + \ [\quad] \ ^{-}}$$

Note that $_1(\ )$ can be seen as an FIR filter and the time-domain representation of $_2(\ )$ is given by:

$$[\ ] = [\ ] - \ [1] \ [\quad - 1] - \ [2] \ [\quad - 2] - \quad - \ [\quad] \ [\quad - \quad]$$

Direct form I can be viewed as $(\ )$ followed by $\frac{1}{(\ )}$.

DIRECT FORM II / TRANSPOSED FORMS

Direct form II implements $\frac{1}{(\ )}$ followed by $(\ )$

Transposed Forms

It is also possible to convert any structure into an equivalent transposed form. This is achieved in the following way:

1: Reverse direction of each interconnection

2: Reverse direction of each multiplier

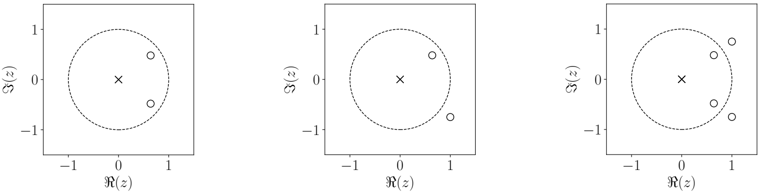
3: Change junctions to adders and vice-versa

4: Interchange the input and output signals

Check: A valid structure must never have any feedback loops that don't go through a delay ($^{-1}$ block).

An FIR filter ( ) is determined by the zeros of

$$(\ ) = \sum_{=0} [\quad - \quad]$$



1 2 3

Which FIR filter has symmetric coefficients [ ]?

A: 1 and 3

B: 2 and 3

C: 3

ANSWER

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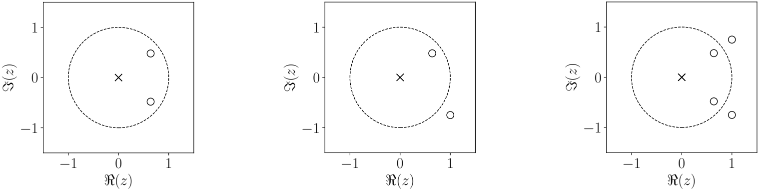

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Which FIR filter has symmetric coefficients [ ]?

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EXPLANATION

Symmetric properties:

Real [ ] conjugate zero pairs:

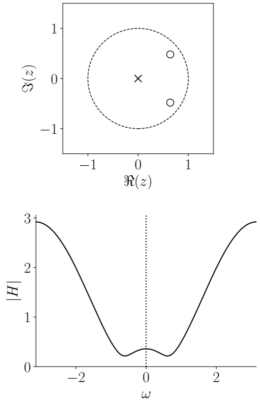
Symmetric: [ ] = [−] reciprocal zero pairs: $^{-1}$

Real & Symmetric [ ] conjugate and reciprocal groups of four (else pairs on the real axis)

EXPLANATION

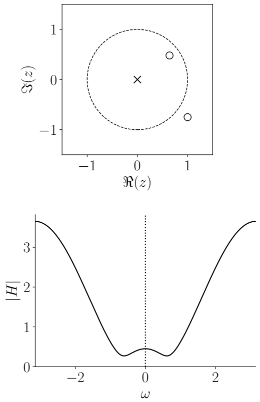
Real

[1, -1.28, 0.64]



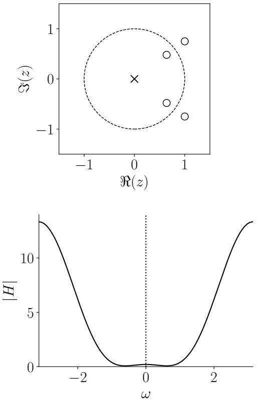
Symmetric

[1, -1.64 + 0.27j, 1]



Real & Symmetric

[1,-3.28, 4.7625, -3.28, 1]



EXPLANATION

In all of the proofs below, we assume that $\ = \ _0$ is a root of $\ (\ )$ so that $\ (\ _0) = \sum_{\ =0} [\ ]\ _0^{-}\ = 0$ and then we prove that this implies that other values of also satisfy $\ (\ ) = 0$.

(1) Real $[\ ]$

$$
\begin{aligned}
(\ _0) &= \sum_{\ =0} [\ ](\ _0)^{-} & \\
&= \sum_{\ =0} [\ ](\ _0)^{-} & \text{sine } [\ ] \text{ is real} \\
&= (\sum_{\ =0} [\ ]\ _0^{-}\ ) & \text{take complex conjugate} \\
&= 0\ \ = 0 & \text{since } (\ _0) = 0
\end{aligned}
$$

EXPLANATION

In all of the proofs below, we assume that = $_0$ is a root of ( ) so that ($_0$) = $\sum_{=0}$ [ ] $_0^{-}$ = 0 and then we prove that this implies that other values of also satisfy ( ) = 0.

(2) Symmetric: [ ] = [−]

$$(_0^{-1}) = \sum_{=0} [\ ]\ _0$$

$$= \sum_{=0} [\ \ -\ \ ]\ _0\ ^{-} \qquad \text{substitute} \ = \ -$$

$$= \ _0 \sum_{=0} [\ \ -\ \ ]\ _0^{-} \qquad \text{take out } _0 \text{ factor}$$

$$= \ _0 \sum_{=0} [\ ]\ _0^{-} \qquad \text{since } [\ \ -\ \ ] = [\ ]$$

$$= \ _0 \times 0 = 0 \qquad \text{since } (_0) = 0$$

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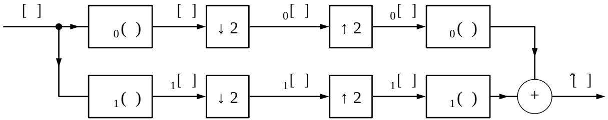
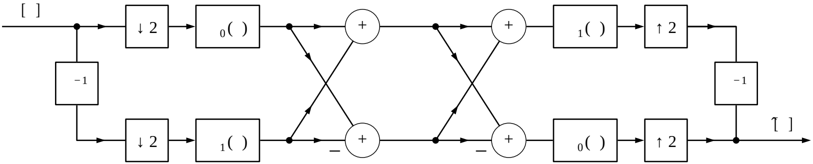
Consider the following Quadrature Mirror Filterbank (QMF):



Which can be implemented using polyphase QMF:



What is the computational saving achieved by the polyphase QMF?

A: A factor of 2

B: A factor of 4

C: A factor of 8

ANSWER

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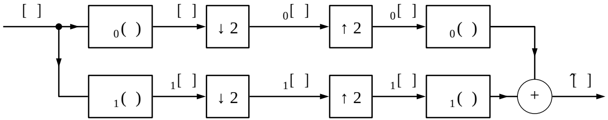

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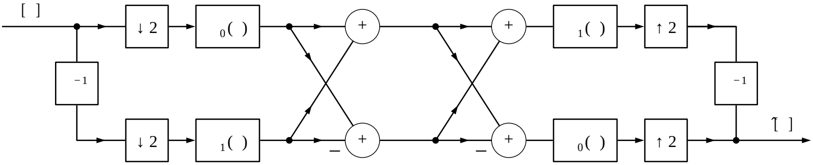

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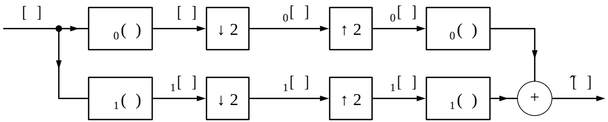
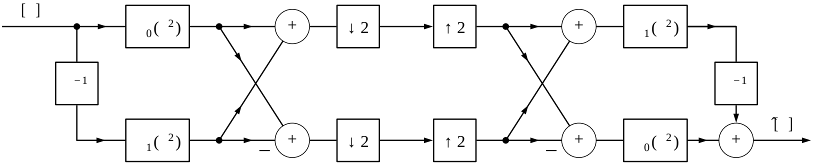
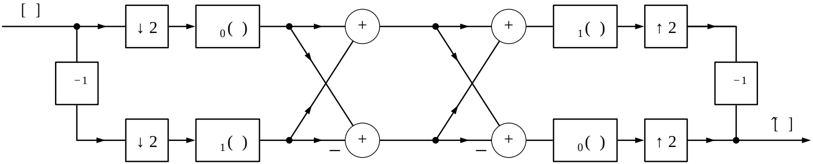
EXPLANATION







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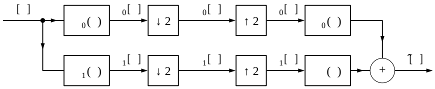
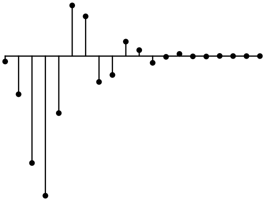
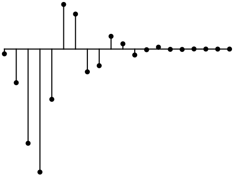


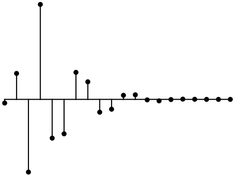


Impulse response ${}_0[\;\;]$

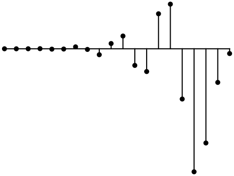
Which is the correct impulse response for ${}_1[\;\;]$?



A



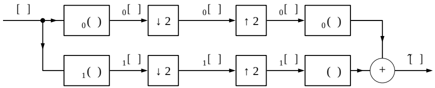
B

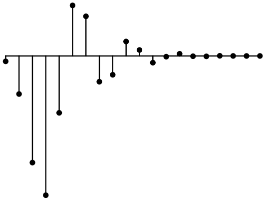


C

ANSWER

Consider the following Quadrature Mirror Filterbank (QMF):





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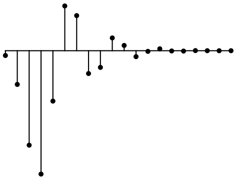
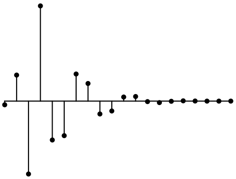
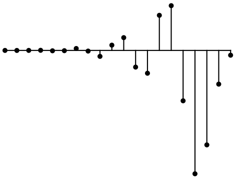


A



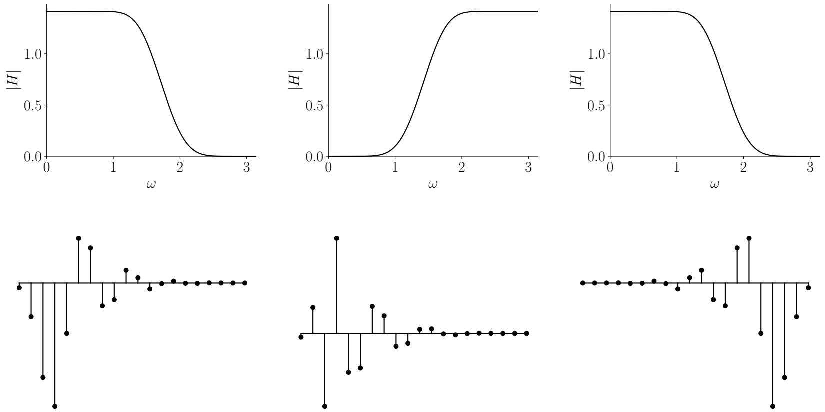
B



C

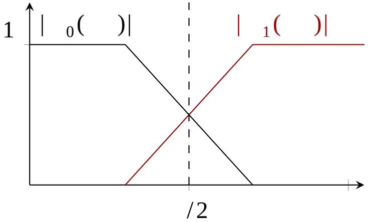
EXPLANATION

Magnitude responses:



$_1[\quad] = \;_0[\quad]$ $\qquad$ $_1[\quad] = (-1) \;_0[\quad]$ $\qquad$ $_1[\quad] = \;_0[\quad - 1 - \quad]$

Proof:



The relationship between $_0(\ )$ and $_1(\ )$ when they are quadrature mirror filters is $_1(\ ) = \ _0(-\ )$.

The corresponding relationship between the impulse responses of these filters is $_1[\ ] = (-1)\ \ _0[\ ]$.

$$_1(\ ) = \ _0(-\ ) = \sum_{=-\infty}^{\infty} \ _0[\ ](-\ )^{-} = \sum_{=-\infty}^{\infty} \ _0[\ ](-1)\ \ ^{-}$$

NEXT WEEK

There will be a class next Thursday (Week 9 - 28/11/2019)

Room: 403A/B

Time: 15:00 to 16:00