Imperial College London

MODULE 4 CLASS

Aidan Hogg - 7 November 2019
ELEC96010 (EE3-07): Digital Signal Processing
Department of Electrical and Electronic Engineering

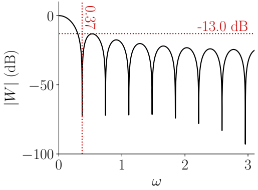
Method:

1: Conceptual question posed - students think quietly on their own and report initial answers on Mentimeter **(3 mins)**

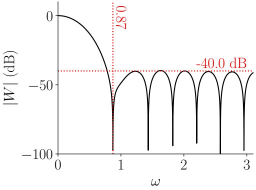
2: Students discuss their answers in small groups **(2 mins)**

3: Explanation/discussion of correct answer **(3 mins)**

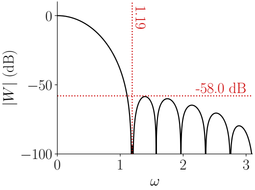
Consider the following windows:



Rectangular



Hamming



Blackman-Harris

If we were to design a filter using the windowing method, which window should be selected if you only care about the transition bandwidth being short?

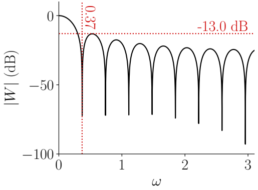
A: Rectangular

B: Hamming

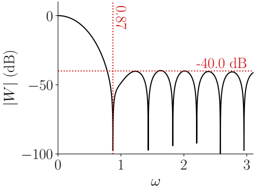
C: Blackman-Harris

ANSWER

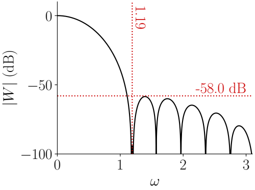
Consider the following windows:



Rectangular



Hamming



Blackman-Harris

If we were to design a filter using the windowing method, which window should be selected if you only care about the transition bandwidth being short?

A: Rectangular

B: Hamming

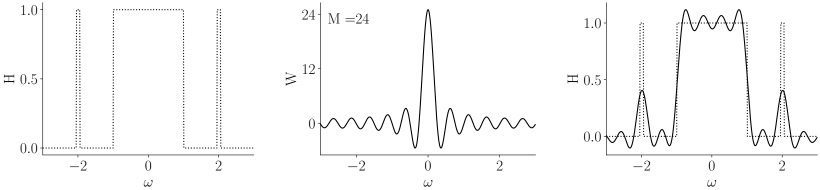
C: Blackman-Harris

EXPLANATION

Window Relationships

Relationship when you multiply an impulse response $h[n]$ by a window $w[n]$ that is of length N

$$H_N(e^{j\omega}) = \frac{1}{2\pi} H(e^{j\omega}) \circledast W(e^{j\omega})$$



Transition bandwidth, $\Delta\omega =$ width of the main lobe

Rectangular window: $\Delta\omega = \frac{4\pi}{N}$

Recall that a digital system in general is described by a difference equation where a recursive difference equation uses past outputs while a non recursive difference equation does not.

Consider the following statements:

1: FIR filters can only be implemented non recursively in practice

2: IIR filters can only be implemented recursively in practice

Which of these statements are true?

A: 1

B: 2

C: Both 1 and 2

ANSWER

Recall that a digital system in general is described by a difference equation where a recursive difference equation uses past outputs while a non recursive difference equation does not.

Consider the following statements:

1: FIR filters can only be implemented non recursively in practice

2: IIR filters can only be implemented recursively in practice

Which of these statements are true?

A: 1

B: 2

C: Both 1 and 2

EXPLANATION

Consider this FIR filter described by a non recursive difference equation:

$$
\begin{aligned}
y[n] &= x[n] + x[n-1] + \cdots + x[n-50] && \text{(a)} \\
y[n-1] &= x[n-1] + x[n-2] + \cdots + x[n-51] && \text{(b)} \\
y[n] - y[n-1] &= x[n] - x[n-51] && \text{(a) - (b)} \\
y[n] &= y[n-1] + x[n] - x[n-51] && \textbf{Recursive}
\end{aligned}
$$

(a) FIR filters are normally implemented non recursively but can be implemented recursively.

(b) IIR filters can only be implemented recursively in practice because an infinite number of coefficients would be required to realize them non recursively (recall: $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$).

The group delay τ_H is measured in what units?

A: Seconds

B: Radians

C: Radians per second

ANSWER

The group delay τ_H is measured in what units?

A: Seconds

B: Radians

C: Radians per second

The group delay is defined $\tau_H(e^{jw}) = -\frac{d\angle H(e^{j\omega})}{d\omega}$

One trick to get at the phase is: $\ln H(e^{j\omega}) = \ln|H(e^{j\omega})| + j\angle H(e^{j\omega})$

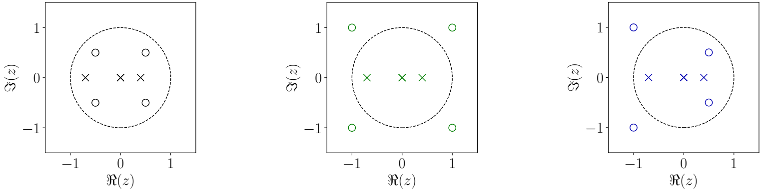
$$\tau_H = \frac{-d(\Im(\ln H(e^{j\omega})))}{d\omega} = \Im\left(\frac{-1}{H(e^{j\omega})}\frac{dH(e^{j\omega})}{d\omega}\right) = \Re\left(\frac{-z}{H(z)}\frac{dH(z)}{dz}\right)\Bigg|_{z=e^{j\omega}}$$

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} h[n]e^{-jn\omega} = \mathcal{F}(h[n]), \qquad \text{where } \mathcal{F} \text{ denotes the DTFT.}$$

$$\frac{dH(e^{j\omega})}{d\omega} = \sum_{n=0}^{\infty} -jnh[n]e^{-jn\omega} = -j\mathcal{F}(nh[n])$$

$$\tau_H = \Im\left(\frac{-1}{H(e^{j\omega})}\frac{dH(e^{j\omega})}{d\omega}\right) = \Re\left(\frac{\mathcal{F}(nh[n])}{\mathcal{F}(h[n])}\right) = \Im\left(j\frac{\mathcal{F}(nh[n])}{\mathcal{F}(h[n])}\right)$$

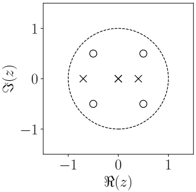
Consider the pole-zero plot of these three systems:



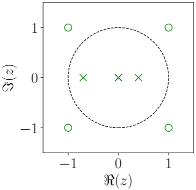
A B C

Which system has most of its energy in $h[n]$ concentrated towards $n = 0$?

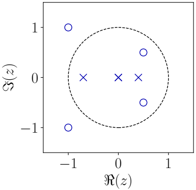
Consider the pole-zero plot of these three systems:



A



B

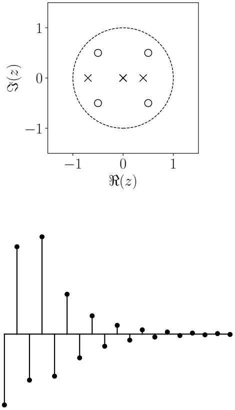


C

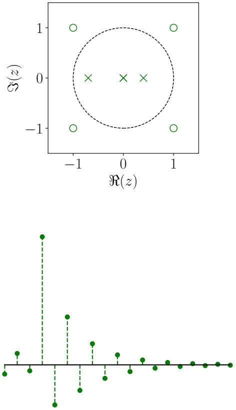
Which system has most of its energy in $h[n]$ concentrated towards $n = 0$?

EXPLANATION

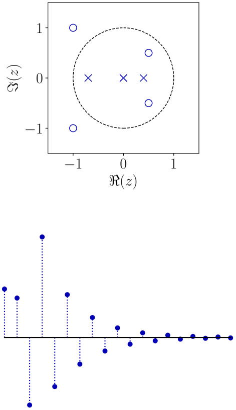
Impulse responses:



A (Minimum Phase)



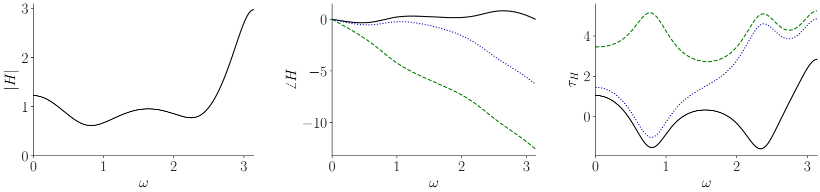
B (Maximum Phase)



C

Minimum Phase

Average group delay over ω equals the (#poles - #zeros) within the unit circle where zeros on the unit circle count for $\frac{1}{2}$



A filter with all its zeros inside the unit circle is a minimum phase filter:

1. The lowest possible group delay for a given magnitude response
2. **The energy in $h[n]$ is concentrated towards $n = 0$**

A Finite Impulse Response (FIR) filter is one where $Y(z) = B(z)X(z)$.

Consider four filters with the following coefficients:

1: $b_1[n] = \{1,\ 0,\ 1,\ 0,\ 0,\ 1,\ 0,\ 1\}$

2: $b_2[n] = \{1,\ 1,\ 1,\ 0,\ 0,\ -1,\ -1,\ -1\}$

3: $b_3[n] = \{1,\ 1,\ 0,\ 1,\ 1,\ 1,\ 1,\ 0,\ 1\}$

4: $b_4[n] = \{0,\ 1,\ 0,\ 1,\ 0,\ 0,\ -1,\ 0,\ -1\}$

Which of these filters will have linear phase?

A: $b_1[n]$

B: Both $b_1[n]$ and $b_2[n]$

C: Both $b_3[n]$ and $b_4[n]$

ANSWER

A Finite Impulse Response (FIR) filter is one where $Y(z) = B(z)X(z)$.

Consider four filters with the following coefficients:

1: $b_1[n] = \{1,\ 0,\ 1,\ 0,\ 0,\ 1,\ 0,\ 1\}$

2: $b_2[n] = \{1,\ 1,\ 1,\ 0,\ 0,\ -1,\ -1,\ -1\}$

3: $b_3[n] = \{1,\ 1,\ 0,\ 1,\ 1,\ 1,\ 1,\ 0,\ 1\}$

4: $b_4[n] = \{0,\ 1,\ 0,\ 1,\ 0,\ 0,\ -1,\ 0,\ -1\}$

Which of these filters will have linear phase?

A: $b_1[n]$

B: Both $b_1[n]$ and $b_2[n]$

C: Both $b_2[n]$ and $b_3[n]$

Linear Phase Filters

The phase of a linear phase filter is: $\angle H(e^{j\omega}) = \theta_0 - \alpha$

Thus the group delay is constant: $\tau_H = -\frac{d\angle H(e^{j\omega})}{d\omega} = \alpha$

A filter has linear phase, if and only if, its impulse response $h[n]$ is symmetric or antisymmetric:

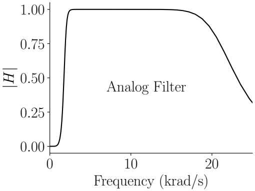
$h[n] = h[N-1-n] \quad \forall n$ or else $h[n] = -h[N-1-n] \quad \forall n$

N can be odd ($\Rightarrow\ni$ mid point) or even ($\Rightarrow\not\ni$ mid point)

Important: This is not the same symmetry that is needed to make the signal real in the frequency domain, which is when $h[n] = h[-n]$.

It is possible to transform a continuous time filter into a discrete time filter using IIR filter transformations.

Which transformation would be most appropriate to transform this continuous time bandpass filter into a discrete time filter?



A: Bilinear mapping

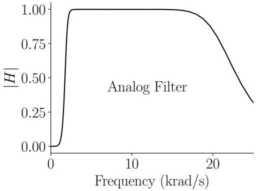
B: Impulse invariance

C: Both bilinear mapping and impulse invariance would be appropriate

ANSWER

It is possible to transform a continuous time filter into a discrete time filter using IIR filter transformations.

Which transformation would be most appropriate to transform this continuous time bandpass filter into a discrete time filter?



A: Bilinear mapping

B: Impulse invariance

C: Both bilinear mapping and impulse invariance would be appropriate

Bilinear Mapping

The bilinear transform is a widely used one-to-one invertible mapping. It involves a change variable:

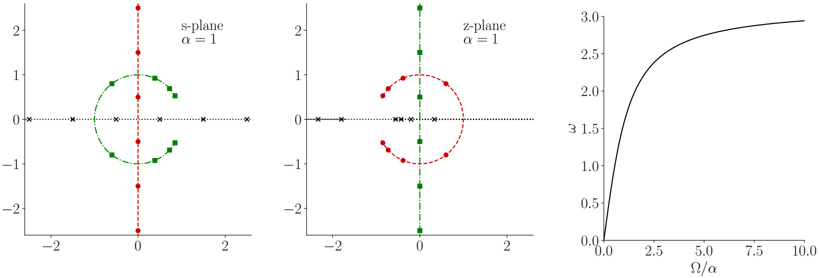
$$z = \frac{\alpha + s}{\alpha - s} \Leftrightarrow s = \alpha \frac{z-1}{z+1}$$

(a) $\Re$ axis $(s) \leftrightarrow \Re$ axis (z)

(b) $\Im$ axis $(s) \leftrightarrow$ Unit circle (z)

(c) Left half plane $(s) \leftrightarrow$ inside of unit circle (z)

(d) Unit circle $(s) \leftrightarrow \Im$ axis (z)



Impulse Invariance

Bilinear transform works well for a low-pass filter but the non-linear compression of the frequency distorts any other response. The impulse invariance transformation is an alternative method that obtains the discrete-time filter by sampling the impulse response of the continuous-time filter.

$$H(s) \xrightarrow{\mathcal{L}^{-1}} h(t) \xrightarrow{\text{sample}} h[n] = T \times h(nT) \xrightarrow{z} H(z)$$

EXPLANATION

Bilinear Mapping

1: Frequency response is identical in both magnitude and phase
2: The frequency axis has a non-linear distortion

Impulse Invariance

1: Preserves frequency axis and impulse response
2: The frequency response is aliased

The question relates to a standard telephone (300 to 3400 Hz bandpass) filter