## Imperial College London

## **MODULE 2 CLASS**

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#### Method:

- 1: Conceptual question posed students think quietly on their own and report initial answers on Mentimeter (3 mins)
- 2: Students discuss their answers in small groups (2 mins)
- 3: Explanation/discussion of correct answer (3 mins)

A sine wave of frequency F is sampled 8 times at a sampling frequency of  $F_s$ .



If the plot shows the magnitude response of the 8 samples what where the values of F and  $F_s$ ?

- A: F = 2000Hz  $F_s = 16000$ Hz
- B: F = 1000Hz  $F_s = 6000$ Hz
- C: F = 2000Hz  $F_s = 8000$ Hz

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#### **EXPLANATION**

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# What is the effect on the frequency spectrum if a continuous signal x(t) is sampled?

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- B: The spectrum becomes real
- C: The spectrum becomes periodic and symmetric

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One Domain	Other Domain
Discrete	Periodic
Symmetric	Symmetric
Antisymmetric	Antisymmetric
Real	Conjugate Symmetric
Imaginary	Conjugate Antisymmetric
Real & Symmetric	Real & Symmetric
Real & Antisymmetric	Imaginary & Antisymmetric

Which x[n] will have a purely real X[k]?

A: 
$$x[n] = \{0, 1, 1, 0, 0, 0, -1, -1\}$$
  
B:  $x[n] = \{1, 1, 0, 1, 1, 0, 1, 1\}$   
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## Which x[n] will have a purely real X[k]?

A: 
$$x[n] = \{0, 1, 1, 0, 0, 0, -1, -1\}$$
  
(X[k] will be purely imaginary)  
B:  $x[n] = \{1, 1, 0, 1, 1, 0, 1, 1\}$   
(X[k] will be complex)  
C:  $x[n] = \{1, 1, 1, 0, 1, 0, 1, 1\}$ 

(X[k]will be purely real)

#### Is it ever a bad idea to window a signal before taking the DFT?

A: Yes

B: No

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A: Yes

B: No

#### **EXPLANATION**

Taking the DFT of a sine waves with an incomplete last cycle

Taking the DFT of a sine waves with an

integer number of

cvcles



It is a bad idea to window a signal before taking the DFT if you have a signal with a complete number of cycles i.e there is no discontinuity to remove in the time domain. (**NEVER the case in real life!**)





#### What is the effect in X[k] of zero padding x[n]?

- A: Denser frequency sampling
- B: Higher frequency resolution
- C: Denser frequency sampling with higher frequency resolution

Zero padding is the process of added extra zeros onto the end of x[n] before performing the DFT.



What is the effect in X[k] of zero padding x[n]?

- A: Denser frequency sampling
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- C: Denser frequency sampling with higher frequency resolution



Zero-padding causes the DFT to evaluate the DTFT at more values of  $\omega_k$ . Denser frequency samples.

Smoother graph but the increased frequency resolution is an **illusion**.

The DFT is defined as: 
$$X[k] = \sum_{0}^{N-1} x[n] e^{-j\frac{2\pi k}{N}n} = \sum_{0}^{N-1} x[n] W_N^{nk}.$$

The FFT is an algorithm that computes the DFT of x[n] using only  $\frac{N}{2} \log_2 N$  complex multiplications.

What is the main property that the FFT exploits to achieve this reduction?

A: 
$$W_N^{2r} = W_{N/2}^r$$
  
B:  $W_N^{r+(N/2)} = -W_N^r$   
C:  $W_N^{kN} = 1$ 

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The DFT matrix of  $F_N$  for N=4. Where  $w=e^{-j\frac{2\pi}{N}}=e^{-j\frac{2\pi}{4}}=-j$ .

$$\text{DFT Matrix: } F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w & w^2 & w^3 \\ 1 & w^2 & w^4 & w^6 \\ 1 & w^3 & w^6 & w^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & (-j) & (-j)^2 & (-j)^3 \\ 1 & (-j)^2 & (-j)^4 & (-j)^6 \\ 1 & (-j)^3 & (-j)^6 & (-j)^9 \end{bmatrix}$$

The matrix has  $N^2$  entries so we would normally have to perform  $N^2$  separate multiplications. However, we can do better! The key to the idea is to connect  $F_N$  with the half-size DFT matrix  $F_{N/2}$ .

#### EXPLANATION

#### Key idea:

$$F_{4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \text{ and } \begin{bmatrix} F_{2} & 0 \\ 0 & F_{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ & 1 & 1 \end{bmatrix}$$
$$F_{4} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The permutation matrix on the right puts  $x_0$  and  $x_2$  (evens) ahead of  $x_1$  and  $x_3$  (odds). The matrix in the middle performs separate half-size transforms on the even and odds. The matrix on the left combines the two half-size outputs to get the correct full-size out.

The same idea applies when N = 1024:

$$F_{1024} = \begin{bmatrix} I_{512} & D_{512} \\ I_{512} & -D_{512} \end{bmatrix} \begin{bmatrix} F_{512} & 0 \\ 0 & F_{512} \end{bmatrix} \begin{bmatrix} \text{even-odd} \\ \text{permutations} \end{bmatrix}$$

 $I_{512}$  is the identity matrix.  $D_{512}$  is the diagonal matrix with entries  $(1,w,...,w^{511}).$ 

**FFT Recursion:** we reduced for  $F_N$  to  $F_{N/2}$ . So lets keep going to  $F_{N/4}$ . The two copies of  $F_{512}$  lead to four copies of  $F_{256}$ . This is the recursion.

$$\begin{bmatrix} I_{256} & D_{256} & & \\ I_{256} & -D_{256} & & \\ & & I_{256} & D_{256} \\ & & & I_{256} & -D_{256} \end{bmatrix} \begin{bmatrix} F_{256} & & \\ & F_{256} & & \\ & & & F_{256} & \\ & & & & F_{256} \end{bmatrix} \begin{bmatrix} \text{pick } 0,4... \\ \text{pick } 2,6... \\ \text{pick } 1,5... \\ \text{pick } 3,7... \end{bmatrix}$$

The reasoning behind  $\frac{1}{2}NL$ . There are L levels, going from  $N = 2^L$  down to N = 1. Each level has  $\frac{1}{2}N$  multiplications from the diagonal D to reassemble the half-size outputs.

This yields the final count  $\frac{1}{2}NL$ , which is  $\frac{1}{2}N\log_2 N$ .