## Imperial College London

## MODULE 2 CLASS

Aidan Hogg - 17 October 2019
ELEC96010 (EE3-07): Digital Signal Processing
Department of Electrical and Electronic Engineering

## PEER INSTRUCTION

## Method:

1: Conceptual question posed - students think quietly on their own and report initial answers on Mentimeter (3 mins)
2: Students discuss their answers in small groups (2 mins)
3: Explanation/discussion of correct answer (3 mins)

A sine wave of frequency $F$ is sampled 8 times at a sampling frequency of $F_{s}$.


If the plot shows the magnitude response of the 8 samples what where the values of $F$ and $F_{s}$ ?

A: $F=2000 \mathrm{~Hz} \quad F_{s}=16000 \mathrm{~Hz}$
B: $F=1000 \mathrm{~Hz} \quad F_{s}=6000 \mathrm{~Hz}$
C: $F=2000 \mathrm{~Hz} \quad F_{s}=8000 \mathrm{~Hz}$

## ANSWER

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## EXPLANATION

A: $F=2000 \mathrm{~Hz} \quad F_{s}=16000 \mathrm{~Hz}$

B: $F=1000 \mathrm{~Hz} \quad F_{s}=6000 \mathrm{~Hz}$

C: $F=2000 \mathrm{~Hz} \quad F_{s}=8000 \mathrm{~Hz}$


What is the effect on the frequency spectrum if a continuous signal $x(t)$ is sampled?

A: The spectrum becomes periodic
B: The spectrum becomes real
C: The spectrum becomes periodic and symmetric

## ANSWER

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| One Domain | Other Domain |
| :--- | :--- |
| Discrete | Periodic |
| Symmetric | Symmetric |
| Antisymmetric | Antisymmetric |
| Real | Conjugate Symmetric |
| Imaginary | Conjugate Antisymmetric |
| Real \& Symmetric | Real \& Symmetric |
| Real \& Antisymmetric | Imaginary \& Antisymmetric |

Which $x[n]$ will have a purely real $X[k]$ ?

$$
\begin{aligned}
& \text { A: } x[n]=\{0,1,1,0,0,0,-1,-1\} \\
& \text { B: } x[n]=\{1,1,0,1,1,0,1,1\} \\
& \text { C: } x[n]=\{1,1,1,0,1,0,1,1\}
\end{aligned}
$$

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$$

## EXPLANATION

Which $x[n]$ will have a purely real $X[k]$ ?

$$
\mathrm{A}: x[n]=\{0,1,1,0,0,0,-1,-1\}
$$

( $X[k]$ will be purely imaginary)
B: $x[n]=\{1,1,0,1,1,0,1,1\}$

$$
\text { ( } X[k] \text { will be complex) }
$$

$\mathrm{C}: x[n]=\{1,1,1,0,1,0,1,1\}$
( $X[k]$ will be purely real)

## go TO WWW.MENTI.COM AND USE THE CODE 767636

Is it ever a bad idea to window a signal before taking the DFT?

A: Yes
B: No

## ANSWER

Is it ever a bad idea to window a signal before taking the DFT?

A: Yes
B: No

## EXPLANATION

Taking the DFT of a sine waves with an incomplete last cycle


DFT after windowing




Taking the DFT of a sine waves with an integer number of cycles

It is a bad idea to window a signal before taking the DFT if you have a signal with a complete number of cycles i.e there is no discontinuity to remove in the time domain. (NEVER the case in real life!)

Zero padding is the process of added extra zeros onto the end of $x[n]$ before performing the DFT.


What is the effect in $X[k]$ of zero padding $x[n]$ ?
A: Denser frequency sampling
B: Higher frequency resolution
C: Denser frequency sampling with higher frequency resolution

## ANSWER

Zero padding is the process of added extra zeros onto the end of $x[n]$ before performing the DFT.


What is the effect in $X[k]$ of zero padding $x[n]$ ?
A: Denser frequency sampling
B: Higher frequency resolution
C: Denser frequency sampling with higher frequency resolution

## EXPLANATION



Zero-padding causes the DFT to evaluate the DTFT at more values of $\omega_{k}$. Denser frequency samples.

Smoother graph but the increased frequency resolution is an illusion.

The DFT is defined as: $X[k]=\sum_{0}^{N-1} x[n] e^{-j \frac{2 \pi k}{N} n}=\sum_{0}^{N-1} x[n] W_{N}^{n k}$.
The FFT is an algorithm that computes the DFT of $x[n]$ using only $\frac{N}{2} \log _{2} N$ complex multiplications.

What is the main property that the FFT exploits to achieve this reduction?

A: $W_{N}^{2 r}=W_{N / 2}^{r}$
B: $W_{N}^{r+(N / 2)}=-W_{N}^{r}$
C: $W_{N}^{k N}=1$

## ANSWER

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## EXPLANATION

The DFT matrix of $F_{N}$ for $N=4$. Where $w=e^{-j \frac{2 \pi}{N}}=e^{-j \frac{2 \pi}{4}}=-j$.

$$
\text { DFT Matrix: } F_{4}=\left[\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & w & w^{2} & w^{3} \\
1 & w^{2} & w^{4} & w^{6} \\
1 & w^{3} & w^{6} & w^{9}
\end{array}\right]=\left[\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & (-j) & (-j)^{2} & (-j)^{3} \\
1 & (-j)^{2} & (-j)^{4} & (-j)^{6} \\
1 & (-j)^{3} & (-j)^{6} & (-j)^{9}
\end{array}\right]
$$

The matrix has $N^{2}$ entries so we would normally have to perform $N^{2}$ separate multiplications. However, we can do better! The key to the idea is to connect $F_{N}$ with the half-size DFT matrix $F_{N / 2}$.

## EXPLANATION

## Key idea:

$$
\begin{gathered}
F_{4}=\left[\begin{array}{cccc}
1 & 1 & 1 & 1 \\
1 & -j & -1 & j \\
1 & -1 & 1 & -1 \\
1 & j & -1 & -j
\end{array}\right] \text { and }\left[\begin{array}{cc}
F_{2} & 0 \\
0 & F_{2}
\end{array}\right]=\left[\begin{array}{cccc}
1 & 1 & & \\
1 & -1 & & \\
& & 1 & 1 \\
& & 1 & -1
\end{array}\right] \\
F_{4}=\left[\begin{array}{cccc}
1 & & 1 & \\
& 1 & & -j \\
1 & & -1 & \\
& 1 & & j
\end{array}\right]\left[\begin{array}{cccc}
1 & 1 & & \\
1 & -1 & & \\
& & 1 & 1 \\
& & 1 & -1
\end{array}\right]\left[\begin{array}{cccc}
1 & & \\
& & 1 & \\
& 1 & & \\
& & & 1
\end{array}\right]
\end{gathered}
$$

The permutation matrix on the right puts $x_{0}$ and $x_{2}$ (evens) ahead of $x_{1}$ and $x_{3}$ (odds). The matrix in the middle performs separate half-size transforms on the even and odds. The matrix on the left combines the two half-size outputs to get the correct full-size out.

## EXPLANATION

The same idea applies when $N=1024$ :

$$
F_{1024}=\left[\begin{array}{cc}
I_{512} & D_{512} \\
I_{512} & -D_{512}
\end{array}\right]\left[\begin{array}{cc}
F_{512} & 0 \\
0 & F_{512}
\end{array}\right]\left[\begin{array}{c}
\text { even-odd } \\
\text { permutations }
\end{array}\right]
$$

$I_{512}$ is the identity matrix. $D_{512}$ is the diagonal matrix with entries $\left(1, w, \ldots, w^{511}\right)$.

FFT Recursion: we reduced for $F_{N}$ to $F_{N / 2}$. So lets keep going to $F_{N / 4}$. The two copies of $F_{512}$ lead to four copies of $F_{256}$. This is the recursion.

$$
\left[\begin{array}{cccc}
I_{256} & D_{256} & & \\
I_{256} & -D_{256} & & \\
& & I_{256} & D_{256} \\
& & I_{256} & -D_{256}
\end{array}\right]\left[\begin{array}{cccc}
F_{256} & & & \\
& F_{256} & & \\
& & F_{256} & \\
& & & F_{256}
\end{array}\right]\left[\begin{array}{l}
\text { pick 0,4... } \\
\text { pick 2,6... } \\
\text { pick 1,5.. } \\
\text { pick 3,7... }
\end{array}\right]
$$

## EXPLANATION

The reasoning behind $\frac{1}{2} N L$. There are $L$ levels, going from $N=2^{L}$ down to $N=1$. Each level has $\frac{1}{2} N$ multiplications from the diagonal $D$ to reassemble the half-size outputs.

This yields the final count $\frac{1}{2} N L$, which is $\frac{1}{2} N \log _{2} N$.

