Exam Corrections: Digital Signal Processing ELEC96010 (EE3-07)

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This document contains all the known corrections to the EE3-07 past papers, dating back to 2010. Please post a question on piazza if there is a mistake in a past paper that has not been listed or if there are errors found within this document.

Exam Paper: Friday, 30 April 2010

Question 5 (d)

The z's are a mistake.

$$g(n) = h(n) + g(n-1)$$

is recursive so...

$$g(2N+2) = h(2N+2)+g(2N+1) = h(2N+2)+h(2N+1)+g(2N) = h(2N+2)+h(2N+1)+h(2N)+g(2N-1) = ...$$

and so on

$$g(2N+2) = \sum_{-\infty}^{2N+2} h[n]$$

$$g(2N+2) = \sum_{0}^{2N+2} h[n] \quad \text{, Due to } h[n] = 0, n < 0$$

$$g(2N+2) = \sum_{0}^{N} h[n] \quad \text{, Due to } h[n] = 0, n > N$$

$$g(2N+2) = a_0 + a_1 + a_2 + ... + a_N$$

Exam Paper: Monday, 9 May 2011

Question 3 (b)

It should be: W_3^{2k}, this would be my answer:

Upsampled spectrum:

$$P(z) = X(z^2)$$

Spectrum after filter:

$$Q(z) = H(z)P(z)$$

Downsampled spectrum:

$$Y(z) = \frac{1}{3}\sum_{k=0}^{2} Q\left(z^{\frac{1}{3}} \times e^{\frac{-j2\pi k}{3}}\right) = \frac{1}{3}\sum_{k=0}^{2} H\left(z^{\frac{1}{3}} \times e^{\frac{-j2\pi k}{3}}\right) P\left(z^{\frac{1}{3}} \times e^{\frac{-j2\pi k}{3}}\right)$$

$$Y(z) = \frac{1}{3}\sum_{k=0}^{2} H\left(z^{\frac{1}{3}} \times e^{\frac{-j2\pi k}{3}}\right) X\left(z^{\frac{2}{3}} \times e^{\frac{-j4\pi k}{3}}\right)$$

$$Y(z) = \frac{1}{3}\sum_{k=0}^{2} H\left(z^{\frac{1}{3}} W_3^{k}\right) X\left(z^{\frac{2}{3}} W_3^{2k}\right)$$

Question 4 (d)

Corrected answer:

$$\begin{aligned}
H(e^{j\omega}) &= \sum_{n=0}^{M-1} h[n]e^{-jn\omega} + \sum_{n=M}^{2M-1} h[n]e^{-jn\omega} \\
&= \sum_{n=0}^{M-1} h[n]e^{-jn\omega} + \sum_{n=0}^{M-1} h[N-1-n]e^{-j(N-1-n)\omega} \\
&= e^{-j(M-0.5)\omega}\Big(\sum_{n=0}^{M-1} h[n]e^{j(M-0.5-n)\omega} + \sum_{n=0}^{M-1} h[N-1-n]e^{-j(M-0.5-n)\omega}\Big) \\
&= e^{-j(M-0.5)\omega}\Big(\sum_{n=0}^{M-1} h[n]\big(e^{j(M-0.5-n)\omega} + e^{-j(M-0.5-n)\omega}\big)\Big) \\
&= e^{-j(M-0.5)\omega}\Big(\sum_{n=0}^{M-1} 2h[n]\cos((M-0.5-n)\omega)\Big) \\
&= e^{j((0.5-M)\omega)}\Big(\sum_{n=0}^{M-1} 2h[n]\cos((M-0.5-n)\omega)\Big)
\end{aligned}$$

Exam Paper: Monday, 21 May 2012

Question 4 (a)

The question is wrong, the correct question is:

A discrete-time system is described by:

$$y(n) = x(n) + x(n-1) - 0.5y(n-2)$$

Therefore the corrected answer is:

n	xa	xb	ya	yb	ya-yb
-2	-1	0	0	0	0
-1	-0.5	0	0	0	0
0	0.5	0.5	0	0.5	-0.5
1	1	1	1.5	1.5	0
2	0.5	0.5	1.5	1.25	0.25
3	-0.5	-0.5	-0.75	-0.75	0
4	-1	-1	-2.25	-2.125	-0.125
5	-0.5	-0.5	-1.125	-1.125	0

However, the figure is correct!

Question 4 (b) part iv

Corrected answer:

$$X(z) = 1 + 2z^{-2}(1 + z^{-1} + z^{-2} + z^{-3} + z^{-5}) - z^{-6}$$

Therefore:

$$x[n] = [1, 0, 2, 2, 2, 2, 1, 2]$$

is correct.

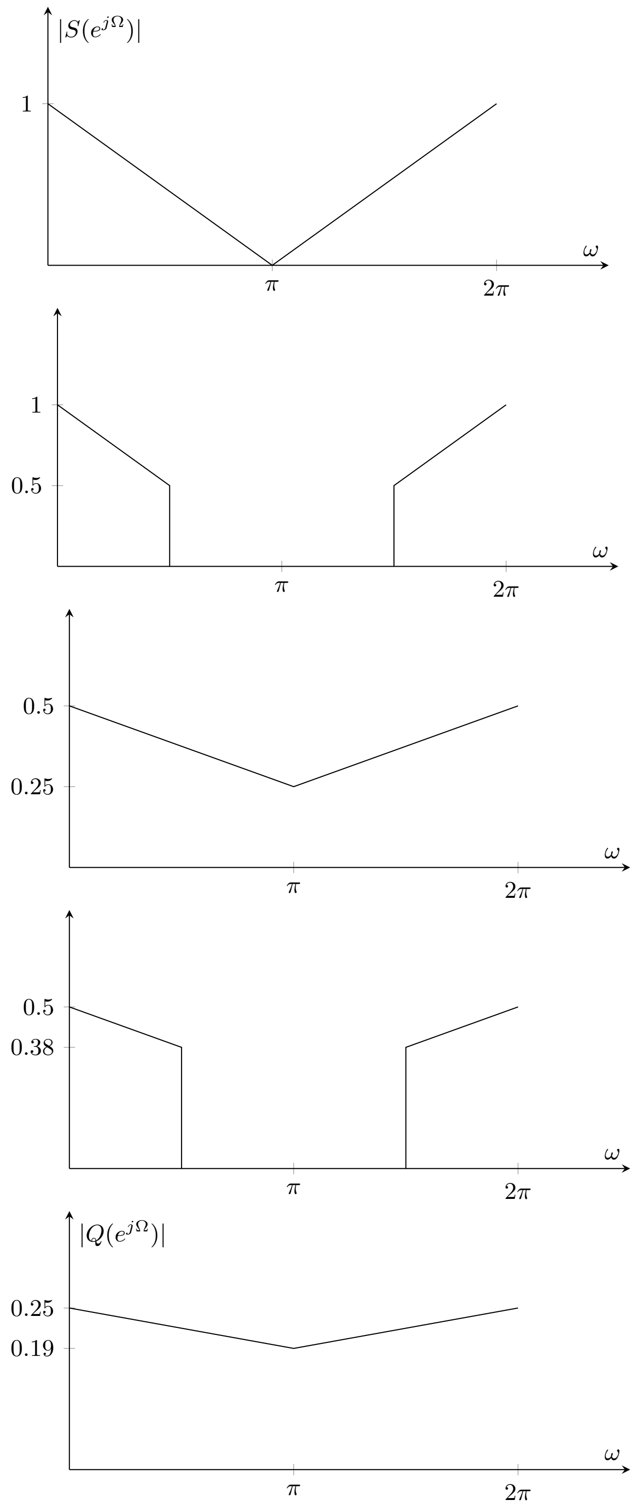
Exam Paper: Wednesday, 9 January 2013

Question 1 (b)

The problem with the question is that it does not state what $H(z)$ is...

If we assume it is a lowpass filter to stop aliasing then the answer is slightly wrong...

Corrected answer:



Question 4 (a) part i

Just to clarify the answer:

$$h[(n-m)_{\text{mod } N}] \equiv h(n-m)_N$$

Question 4 (a) part ii

Corrected answer, the circular convolution will result in:

$$[-5, -16, 7, 18]$$

Question 4 (b) part i

Just to clarify the answer:

$$x[n] = \sum_{n=-\infty}^{\infty} x[n]\delta(t-nT) = \sum_{n=-\infty}^{\infty} x_a(nT)\delta(t-nT) = \sum_{n=-\infty}^{\infty} x_a(t)\delta(t-nT) = x_a(t) \times \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

Exam Paper: Wednesday, 15 January 2014

Question 1 (b) part ii & iii

The question should use linear convolution which becomes multiplication in the frequency domain.

(ii) $x[n] * x[n-2]$

(iii) $x[n] * x[-n]$

Question 4 (c)

W should have subscript $2N$ in the first line...

$$Y[k] = \sum_{n=0}^{2N-1} y[n]e^{-j\frac{2\pi}{2N}nk}$$

Therefore by substitution:

$$Y[k] = \sum_{n=0 \text{ (n even)}}^{2N-1} x[\frac{n}{2}]e^{-j\frac{2\pi}{2N}nk} + \sum_{n=0 \text{ (n odd)}}^{2N-1} 0 \times e^{-j\frac{2\pi}{2N}nk}$$

Which is just:

$$Y[k] = \sum_{n=0 \text{ (n even)}}^{2N-1} x[\frac{n}{2}]e^{-j\frac{2\pi}{2N}nk}$$

Then if we substitute:

$$m = \frac{n}{2}$$

we get:

$$Y[k] = \sum_{m=0}^{N-1} x[m]e^{-j\frac{2\pi}{2N}(2m)k}$$

$$Y[k] = \sum_{m=0}^{N-1} x[m]e^{-j\frac{2\pi}{N}mk} = X[k]$$

However due to the periodicity of the DFT mentioned earlier: (i) $X[k+mN] = X[k]$, where m is an integer (i.e. it repeats every N samples) so the solution is just: $Y[k+mN] = X[k]$, where $k = 0..N-1$ and $m = 0..1$ (Note: $Y[k]$ will repeat every $2N$ samples)

Exam Paper: Monday, 15 December 2014

Question 2 (d)

Corrected answer: $a[1] = -0.4, a[2] = 0.12$

Question 4 (b)

Corrected answer:

Upsampled spectrum:

$$P(e^{j\omega}) = X(e^{jI\omega})$$

Spectrum after filter:

$$Q(e^{j\omega}) = H(e^{j\omega})P(e^{j\omega})$$

Downsampled spectrum:

$$Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} Q\left(e^{\frac{j(\omega-2\pi k)}{D}}\right) = \frac{1}{D}\sum_{k=0}^{D-1} H\left(e^{\frac{j(\omega-2\pi k)}{D}}\right)P\left(e^{\frac{j(\omega-2\pi k)}{D}}\right)$$

$$Y(e^{j\omega}) = \frac{1}{D}\sum_{k=0}^{D-1} H\left(e^{\frac{j(\omega-2\pi k)}{D}}\right)X\left(e^{\frac{j(\omega-2\pi k)I}{D}}\right)$$

Exam Paper: Monday, 14 December 2015

Question 1 (d)

For a causal system

$$Q(z) = \frac{5z + 15.4}{z^2 + 5.2z + 1}$$

Find the inverse z-transform of $Q(z)$ Correct Answer:

$$Q(z) = \frac{5z + 15.4}{z^2 + 5.2z + 1} = \frac{3}{z + 0.2} + \frac{2}{z + 5} = \frac{3z^{-1}}{1 + 0.2z^{-1}} + \frac{2z^{-1}}{1 + 5z^{-1}}$$

Casual system therefore:

$$q(n) = 3(-0.2)^{n-1}u(n-1) + 2(-5)^{n-1}u(n-1)$$

ROC:

$$|z| > 5$$

so the system is unstable because the ROC does not include the unit circle.

P.s. There are many possible solutions because there are many ways to represent the same thing i.e.

$$\begin{aligned}
\delta(n) &= u(n) - u(n-1) \\
\frac{Q(z)}{z} &= \frac{15.4}{z} - \frac{2}{5(z+5)} - \frac{15}{z+0.2} \\
Q(z) &= 15.4 - \frac{2z}{5(z+5)} - \frac{15z}{z+0.2} \\
Q(z) &= 15.4 - \frac{2}{5(1+5z^{-1})} - \frac{15}{1+0.2z^{-1}} \\
q(n) &= 15.4\delta(n) - \frac{2}{5}(-5)^n u(n) - 15(-0.2)^n u(n)
\end{aligned}$$

Question 3 (a) part ii

There should be a second term in the expression for the spectrum of the signal after decimation due to aliasing, therefore, the correct answer is:

$$X(e^{j\omega}) = \frac{2}{1 - ae^{-j\omega}}$$

$$\begin{aligned} Y(e^{j\omega}) &= \frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{\frac{j(\omega-2\pi k)}{M}}\right) = \frac{1}{2}\Big\{X\left(e^{\frac{j\omega}{2}}\right) + X\left(e^{\frac{j(\omega-2\pi)}{2}}\right)\Big\} \\ &= \frac{1}{2}\Big(\frac{2}{1 - ae^{-j\omega/2}} + \frac{2}{1 + ae^{-j\omega/2}}\Big) \end{aligned}$$

Question 4 (c)

Corrected answer:

$$\begin{aligned} &= \sum_{n=0}^{N/2-1} x(n)(-1)^n + \sum_{n=N/2}^{N-1} x(n)(-1)^n \\ &= \sum_{n=0}^{N/2-1} x(n)(-1)^n + \sum_{n=0}^{N/2-1} x(N-1-n)(-1)^{N-1-n} \text{ (reverse order of summation)} \\ &= \sum_{n=0}^{N/2-1} x(n)(-1)^n + \sum_{n=0}^{N/2-1} x(n)(-1)^{N-1-n} \text{ (due to } x(n) = x(N-1-n)) \\ &= \sum_{n=0}^{N/2-1} x(n)(-1)^n + \sum_{n=0}^{N/2-1} x(n)(-1)^N(-1)^{-1}(-1)^{-n} \\ &= \sum_{n=0}^{N/2-1} x(n)(-1)^n - \sum_{n=0}^{N/2-1} x(n)(-1)^n \text{ (due to } (-1)^N = 1, (-1)^{-1} = -1, (-1)^{-n} = (-1)^n) \\ &= 0 \end{aligned}$$

Exam Paper: Wednesday, 14 December 2016

Question 1 (b)

The corrected version of the answer booklet states the group delay is 6 samples which is correct.

Question 3 (c) part ii

The answer book states: “the ‘pole’ at $z = a$ and the ‘zero’ for $k = 0$ cancel out”, so there shouldn’t be a ‘zero’ plotted on the positive real axis in the provided sketch due to cancellation.

Exam Paper: Wednesday, 13 December 2017

Question 1 (b)

The solution is wrong and would only be right if the input was just $2x[n]$. This is because it misses out a boundary condition!

This is the correct answer using the z-transform method:

$$\begin{aligned}
Y(z) &= 0.7z^{-1}Y(z) - 0.1z^{-2}Y(z) + 2X(z) - z^{-2}X(z) \\
(1 - 0.7z^{-1} + 0.1z^{-2})Y(z) &= (2 - z^{-2})X(z) \\
\frac{Y(z)}{X(z)} &= \frac{2 - z^{-2}}{1 - 0.7z^{-1} + 0.1z^{-2}} \\
H(z) &= \frac{2z^2 - 1}{z^2 - 0.7z + 0.1} \\
&= 2 + \frac{1.4z - 1.2}{z^2 - 0.7z + 0.1} \\
&= 2 + \frac{1.4z - 1.2}{(z - 0.2)(z - 0.5)} \\
&= 2 + \frac{\frac{46}{15}}{z - 0.2} - \frac{\frac{5}{3}}{z - 0.5} \\
&= 2 + \frac{46}{15}z^{-1}\frac{1}{1 - 0.2z^{-1}} - \frac{5}{3}z^{-1}\frac{1}{1 - 0.5z^{-1}}
\end{aligned}$$

Thus the impulse response $h[n]$ for the system is:

$$h[n] = 2\delta[n] + \big(\frac{46}{15}(0.2)^{n-1} - \frac{5}{3}(0.5)^{n-1}\big)u[n-1]$$

This is how you would do it correctly using the characteristic equation:

The overall system can be regarded as the cascade of two casual LTI systems:

S1: $y[n] - 0.7y[n-1] + 0.1y[n-2] = x_1[n]$ and S2: $x_1[n] = 2x[n] - x[n-2]$

The impulse response of $h_1[n]$ of the system can be found by solving the complementary solution of $h_1[n] - 0.7h_1[n-1] + 0.1h_1[n-2] = \delta[n]$.

Let the complementary solution be $h_{1c}[n] = \lambda^n$, we have $\lambda^n - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0$ hence $\lambda = \{0.5, 0.2\}$.

Therefore $h_1[n] = h_{1c}[n] = \alpha_1(0.5)^n + \alpha_2(0.2)^n, n \geq 0$.

Solving for α_1 and α_2, we get $\alpha_1 = \frac{5}{3}$ and $\alpha_2 = -\frac{2}{3}$.

Hence $h_1[n] = \frac{5}{3}(0.5)^n - \frac{2}{3}(0.2)^n, n \geq 0$.

The impulse response of $h_2[n]$ of the system S2 is given by $h_2[n] = 2\delta[n] - \delta[n-2]$.

Thus the impulse response $h[n]$ for the overall system is also:

$$h[n] = h_1[n] * h_2[n] = 2\big(\frac{5}{3}(0.5)^n - \frac{2}{3}(0.2)^n\big)u[n] - \big(\frac{5}{3}(0.5)^{n-2} - \frac{2}{3}(0.2)^{n-2}\big)u[n-2]$$

Interestingly you could also solve this equation in the following way:

First workout the complementary solution:

$y_{cs}[n] = \alpha_1(0.5)^n + \alpha_2(0.2)^n, n \geq 0$. (same as above)

Then guess a particular solution that is of a similar form to the input: $y_{ps} = \alpha_3\delta[n]$

Thus the total solution is just $y[n] = y_{cs}[n] + y_{ps}[n]$.

Therefore $y[n] = \alpha_1(0.5)^n + \alpha_2(0.2)^n + \alpha_3\delta[n]$

Now you need to find the three unknowns so set $n = \{0, 1, 2\}$:

$n = 0 : y[0] = \alpha_1 + \alpha_2 + \alpha_3 = 2$

$n = 1 : y[1] = 0.5\alpha_1 + 0.2\alpha_2 + \alpha_3 = 1.4$

$n = 2 : y[2] = 0.25\alpha_1 + 0.04\alpha_2 + \alpha_3 = -0.22$

Solving this set of simultaneous equations gives you:

$\alpha_1 = -\frac{10}{3}, \alpha_2 = \frac{46}{3}, \alpha_3 = -10$

Therefore the impulse response is also:

$$h[n] = -\frac{10}{3}(0.5)^n + \frac{46}{3}(0.2)^n - 10\delta[n]$$

As you can see there are multiple answers...

Why not try and workout the step response yourself (Patrick's solution for the unit step response is of course also wrong).

Question 2 (c)

Corrected answer:

$$X(z) \;—\; \boxed{\downarrow 3} \;—\; \boxed{H(z^{\frac{1}{6}})} \;—\; \boxed{\uparrow 2} \;—\; Y(z)$$

It should be mentioned that $H(z^{\frac{1}{3}})$ is only realisable if $H(z)$ solely contains z's to the powers of 3 (aka $H(z) = 1 + z^{-3} + z^{-6} + z^{-9} + \cdots)$ and likewise $H(z^{\frac{1}{6}})$ is only realisable if $H(z)$ solely contains z's to the powers of 6 (aka $H(z) = 1 + z^{-6} + z^{-12} + z^{-18} + \cdots)$.

The question does not tell us what $H(z)$ is so we can assume that it is possible that $H(z)$ only contains powers of 6.

If $H(z)$ solely contains z's to the powers of k then we would normally write this as $H(z^k)$ to make it clear what powers of z $H(z)$ contains.

You of course should mention this constraint in the exam!

Exam Paper: Monday, 10 December 2018

Question 1 (b)

The answer for 'A' should be $\frac{1}{32}$ instead of 32.