Queen Mary
University of London

May Examination Period 2025

ECS778P Main ***ADVANCED CONTROL SYSTEMS*** Duration: 2 hours (+1 for uploads)

There are FOUR questions on this paper

Answer ALL questions

Each question carries 25% of the marks.

This paper requires **two hours work**. There is an extra hour allowance for downloading the paper and uploading your answers.

You MUST submit your answers before the exam end time.

You must follow the online exam guidelines and instructions on the EECS exam access and submission page.

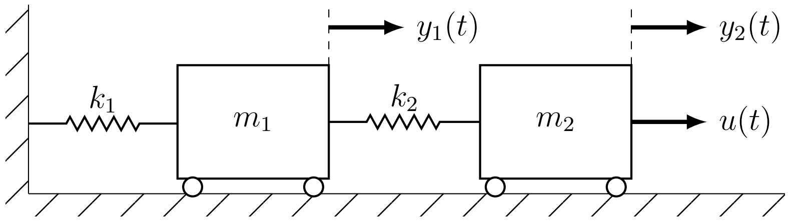
This is an open-book exam. You may use lecture notes and any module materials made available to you (online or physical). You must not use other online resources.

YOU MUST COMPLETE THE EXAM ON YOUR OWN, WITHOUT CONSULTING OTHERS.

Examiners:
Dr. Aidan Hogg and Dr. Kamyar Mehran

Question 1 **[25 marks]**

(a) Consider the translational mechanical system shown below, where $y_1(t)$ and $y_2(t)$ denote the displacements of the associated masses from their static equilibrium positions, and $u(t)$ represents a force applied to the second mass m_2.



The system's parameters are the masses m_1 and m_2 and the spring stiffnesses k_1 and k_2. The input is the applied force $u(t)$, and the outputs are taken as the mass displacements. Given that the state variables are defined as

$$x_1(t) = y_1(t), \quad x_2(t) = y_2(t) - y_1(t), \quad x_3(t) = \dot{y}_1(t), \quad x_4(t) = \dot{y}_2(t).$$

Derive the state-space realization for the mechanical system. That is, derive the coefficient matrices A, B, C, and D.

[9 marks]

(b) Is the following statement True or False:

An observer is a special type of filter designed to estimate the state of a system based on the systems inputs and outputs.

[4 marks]

(c) Is the following statement True or False:

It is possible to make an uncontrollable system controllable using feedback.

[4 marks]

(d) Is the following statement True or False:

For a continuous time system, if all the eigenvalues λ with a positive real part are reachable modes of the system, then the system is stabilizable.

[4 marks]

(e) Is the following statement True or False:

It is possible for a linear system to have two distinct equilibrium points.

[4 marks]

Question 2 **[25 marks]**

Consider the following electrical circuit, which is modelled as a linear, single-input, single-output, continuous-time system of dimension $n = 2$ using the state equations

$$\dot{x}(t) = Ax(t) + Bu(t) \ , \qquad y(t) = Cx(t) + Du(t) \ ,$$

where

$$A = \begin{bmatrix} 0 & \frac{1}{C_1} \\ -\frac{1}{L_1} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \end{bmatrix}$$

and C_1 and L_1 are positive constants representing the circuit's capacitance and inductance, respectively.

(a) Determine the reachability and observability properties of the electrical circuit.

[6 marks]

(b) Express the state-space representation of the electrical circuit in controllable canonical form.

[4 marks]

(c) Let $u(t) = 0$, for all $t \geq 0$. Compute the equilibrium points of the electrical circuit.

[4 marks]

(d) Assume now $u(t) = u(0)$, for all $t \geq 0$, where $u(t) \neq 0$. Compute the equilibrium points of the electrical circuit.

[6 marks]

(e) Determine and justify the Lyapunov stability of the electrical circuit.

[5 marks]

Question 3 **[25 marks]**

Consider a linear, single-input, single-output, discrete-time system of dimension $n = 3$, described by the following state equations

$$x[k+1] = Ax[k] + Bu[k] \ , \qquad y[k] = Cx[k] + Du[k] \ ,$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ , \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ , \quad C = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} \ , \quad D = 1 \ ,$$

and let x_0 be the initial state of the system, that is $x_0 = x[0]$.

(a) Determine and justify whether the system is reachable, controllable, and observable.

[6 marks]

(b) Using the PBH test, determine the unreachable modes of the system.

[3 marks]

(c) Assume $u[0] = 0$. Determine all initial conditions $x[0]$ such that $x[1] = 0$.

[3 marks]

(d) Assume $u[0] = u[1] = 0$. Determine all initial conditions $x[0]$ such that $x[2] = 0$.

[3 marks]

(e) Let $x[0] = 0$. Assume that $y[0] = 0$, $y[1] = 0$ and $y[2] = 1$.

(i) Determine input values $u[0]$, $u[1]$, and $u[2]$ which generate this output sequence.

[6 marks]

(ii) Suppose that the input sequence determined in part (i) is completed with the values $u[k] = 0$, for all $k \geq 3$. Determine the output sequence $y[k]$ for all $k \geq 3$.

[2 marks]

(iii) Explain why the output sequence is constant for all $k \geq 3$.

[2 marks]

Question 4 **[25 marks]**

Consider a linear, single-input, single-output, continuous-time system of dimension $n = 3$ described by the following state equations

$$\dot{x}_1(t) = x_2(t) \ ,$$
$$\dot{x}_2(t) = x_3(t) \ ,$$
$$\dot{x}_3(t) = -\alpha x_3(t) + u(t) \ ,$$

with the output equation

$$y(t) = x_1(t) \ .$$

(a) Show, using the PBH test, that the system is observable for all α.

[5 marks]

(b) Design an asymptotic observer for the system. Select the output injection gain L such that the matrix $A + LC$ has three eigenvalues equal to $\lambda = -2$.

[10 marks]

(c) Design an output feedback controller applying state feedback of gain K such that the matrix $A + BK$ has three eigenvalues equal to $\lambda = -1$.

[10 marks]

End of questions