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## Lecture Exercises (part 2)

**Question 1.** Consider the following discrete-time system

$$x^+ = Ax + Bu$$

where

$$A = \begin{bmatrix} 0 & \alpha \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{for } -1 \leq \alpha \leq 1$$

- (a) Determine the reachability and controllability as a function of  $\alpha$ .
- (b) Study stability properties as a function of  $\alpha$ .
- (c) Design a state feedback control law such that the closed-loop system is asymptotically stable.

**Question 2.** Consider the following continuous-time system

$$\dot{x} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & \alpha \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

- (a) Study the stability properties as a function of  $\alpha$ .
- (b) Show that the system is non-controllable but is stabilizable.
- (c) Design a feedback controller that renders the closed-loop system asymptotically stable for all values of  $\alpha$ .

**Question 3.** Consider the following discrete-time system

$$x[k+1] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} x[k]$$

$$y[k] = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} x[k]$$

- (a) Determine if the system is observable and find any unobservable modes.
- (b) Assume the output of the system is  $y[0] = y[1] = y[2] = 0$
- Compute all possible initial states,  $x[0]$ .
  - Compute all possible final states,  $x[2]$ .
  - Discuss the differences between part (i) and part (ii) and what the results tell you about the system.
- (c) Design an observer which places all eigenvalues at  $\lambda = 0$  where for any  $e[0]$ ,  $e[k] = 0$  for  $k \geq \bar{k}$ . Determine the value of  $\bar{k}$ .