Lecture Exercises (part 2)

Question 1. *Consider the following discrete-time system*

$$x^{+} = Ax + Bu$$

where

$$A = \begin{bmatrix} 0 & \alpha \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \textit{for } -1 \leq \alpha \leq 1$$

(a) Determine the reachability and controllability as a function of α.

(b) Study stability properties as a function of α.

(c) Design a state feedback control law such that the closed-loop system is asymptotically stable.

Question 2. *Consider the following continuous-time system*

$$\dot{x} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & \alpha \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

(a) *Study the stability properties as a function of* α.

(b) *Show that the system is non-controllable but is stabilizable.*

(c) *Design a feedback controller that renders the closed-loop system asymptotically stable for all values of* α.

Question 3. *Consider the following discrete-time system*

$$x[k+1] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} x[k]$$

$$y[k] = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} x[k]$$

(a) *Determine if the system is observable and find any unobservable and reachable modes.*

(b) *Assume the output of the system is* $y[0] = y[1] = y[2] = 0$

(i) *Compute all possible initial states,* $x[0]$.

(ii) *Compute all possible final states,* $x[2]$.

(iii) *Discuss the differences between part (i) and part (i) and what the results tell you about the system.*

(c) *Design and observer which places all eigenvalues at* $\lambda = 0$ *where for any* $e[0], \quad e[k] = 0$ *for* $k \geq \bar{k}$. *Determine the value of* $\bar{k}$.