Lecture Exercises (part 1)

Question 1. *Consider the following electrical circuit, which is modelled as a linear, single-input, single-output, continuous-time system of dimension $n = 3$ using the state equations*

$$
\begin{aligned}
\dot{x}_1 &= \frac{1}{L_1}(u - x_3) && \textit{(current through inductor } L_1\textit{)} \\
\dot{x}_2 &= \frac{1}{L_2}(u - x_3) && \textit{(current through inductor } L_2\textit{)} \\
\dot{x}_3 &= \frac{1}{C}(x_1 + x_2) && \textit{(voltage across the capacitor } C\textit{)} \\
y &= x_1 + x_2
\end{aligned}
$$

(a) Write the state-space representation of the system.

(b) Determine and justify whether the system is controllable.

(c) Determine and justify whether the system is observable.

(d) Find the unobservable subspace and determine physical property of the electrical circuit is non-distinguishable.

(e) Use the PBH test to find the unobservable modes of the system.

(f) Let $\hat{x} = T^{-1}x$ where

$$
T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}
$$

Find the state space representation of the system with the state $\hat{x}$, identify the unobservable subsystem and explain why the system's eigenvalues coincide with the observable modes found in the previous part of the question.

Question 2. *Consider the discrete-time system*

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}, \qquad D = -1$$

(a) Determine and justify whether the system is reachable.

(b) Determine and justify whether the system is observable.

(c) Assume $u[0] = u[1] = u[2] = 1$ *which give the output signal* $y[0] = -5,\ y[1] = 0,\ y[2] = 1$*. Find the free response of the output for* $k = \{0, 1, 2\}$*.*

(d) Find a sequence that drives the initial state $x[0]$ *to zero and what is the smallest number of steps needed.*

Question 3. *Consider the continuous-time system*

$$A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

(a) *Determine and justify whether the system is controllable as a function of* α*.*

(b) *Design an output feedback controller applying state feedback of gain* K *such that the matrix* $A + BK$ *has all its eigenvalues equal to* $\lambda = -1$*.*

(c) *Determine and justify whether the system is observable as a function of* α*.*

(d) *Design an asymptotic observer for the system. Select the output injection gain* L *such that the matrix* $A + LC$ *has all its eigenvalues equal to* $\lambda = -2$*.*