Week 9 Tutorial

Question 1. *Consider the discrete-time system $x(k+1) = Ax(k)$, $y(k) = Cx(k)$. Let*

$$A = \begin{bmatrix} 0 & -4 & 0 \\ 1 & 4 & 0 \\ 0 & -4 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

Determine if the system is observable and compute the unobservable subspace.

Question 2. *Consider the continuous-time system*

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 3 & -1+\epsilon \\ 1 & 2-\epsilon \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} -1 & 1 \end{bmatrix} x. \end{aligned}$$

(a) Show that the system is observable for all $\epsilon \neq 1/2$.

(b) Let $\epsilon = 1/2$. Determine, using PBH test, the unobservable modes.

Question 3. *Consider the linear electrical network in Figure A. Let u be the driving voltage.*

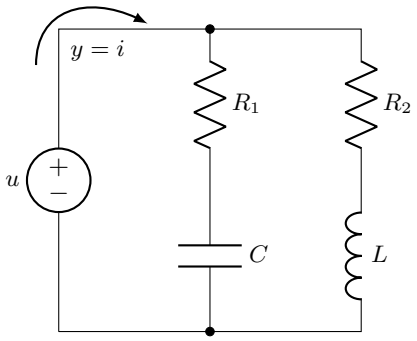

Figure A: The electrical network for Question 3.

(a) Using Kirchhoff's laws, or otherwise, express the dynamics of the circuit in the standard state-space form

$$\dot{x} = Ax + Bu \qquad y = Cx + Du$$

Take x_1 to be the voltage across the capacitor, x_2 to be the current through the inductor and the output to be the current supplied by the generator.

(b) Derive a condition on the parameters R_1, R_2, C and L under which the pair (A, C) is observable.

(c) Assume $R_1 R_2 C = L$. Define the unobservable subspace. Illustrate this subspace as lines in $\mathcal{R}^2$.