

## Week 6 Tutorial

**Question 1.** Consider the discrete-time system  $x[k+1] = Ax[k] + Bu[k]$ . Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

- (a) Compute the reachability matrix  $R$ .
- (b) Determine if the system is reachable and compute the set of reachable states.
- (c) Determine all states  $x_I$  such that  $x[0] = x_I$  and  $x[1] = 0$ .

**Question 2.** Consider the continuous-time system  $\dot{x} = Ax + Bu$ . Let

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

- (a) Compute the reachability matrix  $R$ .
- (b) Determine if the system is reachable.
- (c) Compute the set of states that can be reached from the state,  $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ .

**Question 3.** Consider the discrete-time system  $x[k+1] = Ax[k] + Bu[k]$ . Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

- (a) Compute the reachability matrix  $R$ .
- (b) Determine if the system is reachable.
- (c) Compute the reachable subspaces in one step, two steps and three steps.