Week 3 Tutorial

Question 1. *Consider a linear continuous-time system described by the equations*

$$\begin{aligned}\dot{x}_1(t) &= x_1(t) + \alpha x_2 + u(t)\\ \dot{x}_2(t) &= x_1(t) + x_2(t) - \alpha x_2(t)\\ y(t) &= x_1(t)\end{aligned}$$

with $\alpha \in \mathbb{R}$ *and constant,* $x(t) = [x_1(t), x_2(t)]^T \in \mathbb{R}^2$ *and* $u(t) \in \mathbb{R}$.

1. *Let* $u(t) = 0$*, for all* $t \geq 0$*. Compute the equilibrium points of the system as a function of* α*.*
2. *Assume now* $u(t) = u(0)$*, for all* $t \geq 0$*, where* $u(t) \neq 0$*. Compute the equilibrium points of the system as a function of* α*.*
3. *Discuss similarities and differences between the results in part (a) and part (b).*

Question 2. *An ideal op-amp circuit is given in Figure 1*

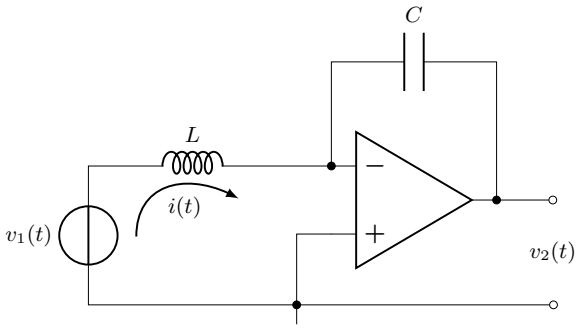

Figure 1

where $i(t)$ *is the current,* $v_1(t)$ *is the input and* $v_2(t)$ *is the output.*

1. *Derive the state space model for the circuit in Figure 1 using the state variables* $x_1 = i(t)$ *and* $x_2 = v_2(t)$*.*
2. *Using your answer from part 1, obtain the transfer function* $G(s)$ *of the circuit in Figure 1.*
3. *Find the state transition matrix* e^{At} *such that* $\mathbf{x}(t) = e^{At}\mathbf{x}(0)$*.*
4. *Find the equilibrium point of the circuit in Figure 1.*