Week 2 Tutorial

Question 1. *Consider the translational mechanical system shown in Figure 1, where $y_1(t)$ and $y_2(t)$ denote the displacements of the associated masses from their static equilibrium positions, and $f(t)$ represents a force applied to the first mass m_1.*

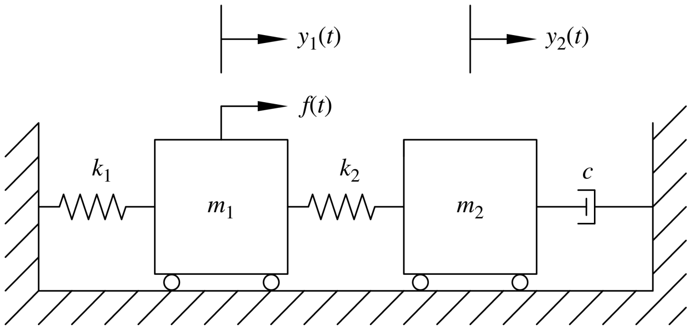

Figure 1: Translational mechanical system.

The system's parameters are the masses m_1 and m_2, viscous damping coefficient c, and spring stiffnesses k_1 and k_2. The input is the applied force $u(t) = f(t)$, and the outputs are taken as the mass displacements.

Derive a valid state-space realization for the mechanical system. That is, specify the state variables and derive the coefficient matrices A, B, C, and D.

Question 2. *Consider the electrical network shown in Figure 2. The two inputs are the independent voltage and current sources $v_{in}(t)$ and $i_{in}(t)$, and the single output is the inductor voltage $v_L(t)$.*

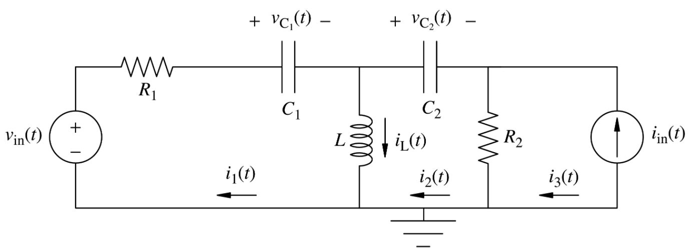

Figure 2: Electrical Circuit.

Using clockwise circulating mesh currents $i_1(t)$, $i_2(t)$, and $i_3(t)$, Kirchhoff's voltage and current laws yield:

$$R_1 i_1(t) + v_{C_1}(t) + L\frac{d}{dt}[i_1(t) - i_2(t)] = v_{in}(t),$$

$$L\frac{d}{dt}[i_2(t) - i_1(t)] + v_{C_2}(t) + R_2[i_2(t) - i_3(t)] = 0,$$

$$i_3(t) = -i_{in}(t),$$

$$i_L(t) = i_1(t) - i_2(t).$$

Derive a valid state-space realization for the electrical network. That is, specify the state variables and derive the coefficient matrices A, B, C, and D.

Question 3. *Consider the matrix*

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

1. *Compute the characteristic polynomial of A and find the eigenvalues of A.*
2. *Compute three linearly independent eigenvectors for A.*
3. *Find a similarity transformation L such that $\hat{A} = L^{-1}AL$ is a diagonal matrix.*
4. *Compute e^{At} as a function of t.*
5. *Compute $\sin(At)$ as a function of t.*