Week 12 Tutorial - Example Exam Question

The final exam for 'Advanced Control Systems' consists of 4 questions, each worth 25 marks. Below is an example question of the type you can expect in the exam.

Question 1. **[25 marks]**

A student's knowledge is modelled using a discrete-time system where the student's knowledge level evolves over time based on their study effort and natural forgetting. The system is described by the following equations

$$M_1[k+1] = M_1[k] - \beta_1 M_1[k] + \gamma E[k] - \alpha M_1[k],$$

$$M_2[k+1] = M_2[k] - \beta_2 M_2[k] + \alpha M_1[k] + \delta M_2[k],$$

where $M_1[k]$ and $M_2[k]$ represent the student's short-term and long-term knowledge, respectively, with the signal $E[k]$ being the student's study effort.

The constants β_1 and β_2 are the forgetting rates for short-term and long-term knowledge, respectively. The constant α represents how effectively short-term knowledge transfers to long-term memory, γ measures how efficiently studying contributes to short-term knowledge, and δ accounts for the reinforcement of long-term memory through review.

The student's study effort is adjusted based on the gap between their perceived knowledge level and their target knowledge goal M_0, modelled as:

$$E[k+1] = \eta(M_0 - M_1[k] - M_2[k]),$$

where η is a parameter governing how strongly the student reacts to their knowledge gap.

(a) Derive the state space model for the student's knowledge using the state variables $x_1 = M_1$, $x_2 = M_2$ and $x_3 = E$, where the target knowledge goal is the input $u = M_0$, and the output is the sum of knowledge, $y = M_1 + M_2$. That is, derive the coefficient matrices A, B, C, and D.

[4 marks]

(b) Assume that the constants for the system are defined as follows: $\beta_1 = 1$, $\beta_2 = 1$, $\alpha = 1$, $\gamma = 0$, $\delta = 2$ and $\eta = 1$.

(i) Determine and justify whether the system is reachable, controllable, and observable.

[6 marks]

(ii) Assume that $u[k] = u[0]$, for all $k \geq 0$, where $u[0] \neq 0$. Compute the equilibrium points of the system.

[5 marks]

(iii) Using the PBH test, determine the unobservable modes of the system.

[5 marks]

(iv) Determine if the system is stable in terms of Lyapunov stability, and if the system is unstable, comment on whether the system is stabilizable using feedback control.

[5 marks]