Week 10 Tutorial

Question 1. *Consider the continuous-time system $\dot{x} = Ax + Bu$. Let*

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find a matrix K such that $P_\lambda(A + BK) = \{-1, -2\}$. Solve the problem in two ways:

(a) *using Ackermann's formula (explained the 'module notes'), which states*

$$K = -\begin{bmatrix} 0 & 1 \end{bmatrix} R^{-1} p(A),$$

such that the characteristic polynomial of $A + BK$ is equal to $p(s)$.

(b) *using a direct computation seen in the lecture, i.e. without computing the reachability matrix of the system.*

Question 2. *Consider the continuous-time system*

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \\ y &= \begin{bmatrix} 3 & -1 \end{bmatrix} x. \end{aligned}$$

(a) *Show that the system is controllable and observable.*

(b) *Design a state feedback control law*

$$u = Kx + v$$

such that the closed-loop system has two eigenvalues at -3.

Question 3. *Consider the continuous-time system*

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 3 & -1+\epsilon \\ 1 & 2-\epsilon \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} -1 & 1 \end{bmatrix} x. \end{aligned}$$

(a) *Show that the system is controllable for any $\epsilon \neq 1$. Study the stabilisability of the system for $\epsilon = 1$.*

(b) *Show that the system is observable for any $\epsilon \neq 1/2$. Study the detectability of the system for $\epsilon = 1/2$.*

(c) *Assume $\epsilon = 0$. Design a state feedback control law*

$$u = Kx + v$$

such that the closed-loop system has two eigenvalues equal to -2.

(d) *Show that the state feedback control law designed above stabilizes the system for any $\epsilon \in (-4, 1/7)$.*