Tutorial Problem Sheet 9

Question 1. *Consider the discrete-time system*

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u_k, \\ y_k &= \begin{bmatrix} 3 & -1 \end{bmatrix} x_k. \end{aligned}$$

(a) Show that the system is observable.

(b) Design an asymptotic observer where both eigenvalues of $A + LC$ are zero and with state $\hat{x}_k$, such that $e_k = x_k - \hat{x}_k = 0$ for all $k \geq N$. Determine the smallest value of N for which the above condition can be satisfied.

(c) Let $u_k = K\hat{x}_k + v_k$ with $K = [3/4, 3/4]$.
Write the equations of the closed-loop system, with state $[x_k, \hat{x}_k]$, input v_k and output y_k, and determine the eigenvalues of this system.

Question 2. *Consider the continuous-time system*

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u, \\ y &= \begin{bmatrix} 3 & -1 \end{bmatrix} x \end{aligned}$$

(a) Design an asymptotic observer with a double pole at -6.

(b) Suppose x_0 is the observer state evaluated in part 1. Let $u = Kx_0 + v$ with $K = [-4, 2]$.
Compute the eigenvalues of the closed-loop system.

Question 3. *Consider the continuous-time system $\dot{x} = Ax$, $y = Cx$. Let*

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

(a) Show, using PBH test, that the system is observable.

(b) Design an asymptotic observer for the system. Select the output injection gain L such that the matrix $A + LC$ has two eigenvalues equal to -3.

(c) Suppose that one can measure $y(t)$ and a delayed copy of $y(t)$ given by $y(t - \tau)$, with $\tau > 0$.

For $t \geq \tau$, express the vector $Y(t) = \begin{bmatrix} y(t) \\ y(t-\tau) \end{bmatrix}$ from $x(0)$.

Show that the relation determined above can be used, for any $\tau > 0$, to compute $x(0)$ as a function of $Y(t)$, where $t \geq \tau$. Argue that the above result can be used to determine $x(t)$ from $Y(t)$, for $t \geq \tau$, exactly.