Tutorial Problem Sheet 5

Question 1. *Consider the linear continuous-time system described by the equations*

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

For each of the following cases, determine the stability properties of the system and justify your conclusion:

(a) *When* $A = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, *what can you conclude about the stability properties of the system?*

(b) *When* $A = \begin{bmatrix} -3 & 4 & -4 \\ 0 & 5 & -1 \\ 0 & 4 & -7 \end{bmatrix}$, *what can you conclude about the stability properties of the system?*

Question 2. *Consider the discrete-time system* $x[k+1] = Ax[k] + Bu[k]$ *from last week. Let*

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

(a) *Using PHB test determine the unreachable modes.*

(b) *Show that the system is controllable.*

Question 3. *Consider the continuous-time system* $x[k+1] = Ax[k] + Bu[k]$ *from last week. Let*

$$A = \begin{bmatrix} 0 & \gamma \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(a) *Show using PHB test that the system is reachable.*

(b) *Express the state-space representation of the continuous-time system in controllable canonical form.*

Question 4. *Consider the discrete-time system* $x[k+1] = Ax[k] + Bu[k]$. *Let*

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(a) *Using PHB test determine the unreachable modes.*

(b) *Comment on whether the system is controllable.*