Tutorial Problem Sheet 4

Question 1. *Consider the linear discrete-time system described by the equations*

$$x^+[k] = Ax[k] + Bu[k]$$
$$y[k] = Cx[k] + Du[k]$$

For each of the following cases, determine the stability properties of the system and justify your conclusion:

(a) *When* $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, *what can you conclude about the stability properties of the system?*

(b) *When* $A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$, *what can you conclude about the stability properties of the system?*

Question 2. *Consider the linear continuous-time system described by the equations*

$$\dot{x}_1 = -x_1$$
$$\dot{x}_2 = -x_1$$
$$\dot{x}_3 = -x_1$$

What can you conclude about the stability of the system?

Question 3. *Consider the discrete-time system* $x[k+1] = Ax[k] + Bu[k]$. *Let*

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

(a) *Compute the reachability matrix* R.

(b) *Determine if the system is reachable and compute the set of reachable states.*

(c) *Determine all states* x_I *such that* $x[0] = x_I$ *and* $x[1] = 0$.

Question 4. *Consider the continuous-time system* $\dot{x} = Ax + Bu$. *Let*

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(a) *Compute the reachability matrix* R.

(b) *Determine if the system is reachable.*

(c) *Compute the set of states that can be reached from the state,* $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Question 5. *Consider the discrete-time system* $x[k+1] = Ax[k] + Bu[k]$. *Let*

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

(a) Compute the reachability matrix R.

(b) Determine if the system is reachable.

(c) Compute the reachable subspaces in one step, two steps and three steps.