Tutorial Problem Sheet 3

Question 1. *Consider the linear continuous-time system from last week described by the equations*

$$\begin{aligned}\dot{x}_1(t) &= x_1(t) + \alpha x_2 + u(t)\\ \dot{x}_2(t) &= x_1(t) + x_2(t) - \alpha x_2(t)\\ y(t) &= x_1(t)\end{aligned}$$

with $\alpha \in \mathbb{R}$ *and constant,* $x(t) = [x_1(t), x_2(t)]^T \in \mathbb{R}^2$ *and* $u(t) \in \mathbb{R}$.

(a) Study the stability properties of the system when $\alpha > 2$. *(Hint: compute the characteristic polynomial of the closed-loop system and then use the Routh test.)*

(b) Let $u(t) = -ky(t)$, *where* k *is a constant. Write the equations of the closed-loop system and determine conditions on* α *and* k *such that the closed-loop system is asymptotically stable.*

Question 2. *Consider the discrete-time system* $x_{k+1} = Ax_k$.

(a) Let

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}.$$

Consider the initial state

$$x[0] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and plot $x[k]$ *on the state space for* $k = 1, 2, 3, 4$. *Exploiting the obtained result discuss the stability of the equilibrium* $x_e = 0$.

(b) Let

$$A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Consider the initial state

$$x[0] = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

and plot $x[k]$ *on the state space for* $k = 1, 2, 3, 4$. *Exploiting the obtained result discuss the stability of the equilibrium* $x_e = 0$.

Question 3. *Consider the discrete-time system*

$$x_{k+1} = Ax_k = \begin{bmatrix} 1 & 1 \\ a & -1 \end{bmatrix} x_k$$

with $a \in \mathbb{R}$.

Show that the system is asymptotically stable for $-2 < a < 0$, *and it is unstable for* $a < -2$ *and* $a > 0$.