Tutorial Problem Sheet 2

Question 1. *An ideal op-amp circuit is given in Figure 1*

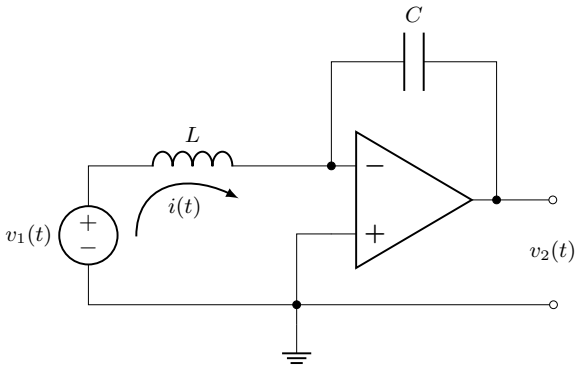

Figure 1

where $i(t)$ is the current, $v_1(t)$ is the input and $v_2(t)$ is the output.

(a) *Derive the state space model for the circuit in Figure 1 using the state variables $x_1 = i(t)$ and $x_2 = v_2(t)$.*

(b) *Using your answer from part (a), obtain the transfer function $G(s)$ of the circuit in Figure 1.*

(c) *Find the state transition matrix e^{At} such that $\mathbf{x}(t) = e^{At}\mathbf{x}(0)$.*

(d) *Find the equilibrium point of the circuit in Figure 1.*

Question 2. *Consider the discrete-time linear system*

$$\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}u[k],$$
$$y[k] = \mathbf{C}\mathbf{x}[k],$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ -\frac{T}{C} & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{T}{L} \\ 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

and $T > 0$ is the sampling period.

(a) *Show that the discrete-time state matrix $\mathbf{A}$ corresponds to the continuous-time system studied in Question 1.*

(b) *Compute $\mathbf{A}^k$ for all $k \geq 0$.*

(c) *Hence, derive an explicit expression for the state trajectory $\mathbf{x}[k]$ in terms of the initial condition $\mathbf{x}[0]$ and the input sequence $\{u[0], u[1], \ldots, u[k-1]\}$.*

(d) *Assume $u[k] = 0$ for all k. Describe the free response of the system.*

(e) *Determine all equilibrium points of the discrete-time system and compare them with the equilibrium points of the continuous-time model.*