Tutorial Problem Sheet 1

Question 1. *Consider the translational mechanical system shown in Figure 1, where $y_1(t)$ and $y_2(t)$ denote the displacements of the associated masses from their static equilibrium positions, and $f(t)$ represents a force applied to the first mass m_1.*

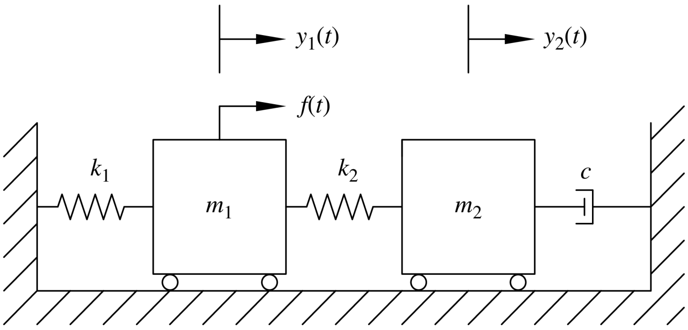

Figure 1: Translational mechanical system.

The system's parameters are the masses m_1 and m_2, the viscous damping coefficient c, and the spring stiffnesses k_1 and k_2. The input is the applied force $u(t) = f(t)$, and the outputs are taken as the mass displacements.

Derive a valid state-space realisation for the mechanical system. That is, specify the state variables and derive the coefficient matrices A, B, C, and D.

Question 2. *Consider the electrical network shown in Figure 2. The two inputs are the independent voltage and current sources $v_{in}(t)$ and $i_{in}(t)$, and the single output is the inductor voltage $v_L(t)$.*

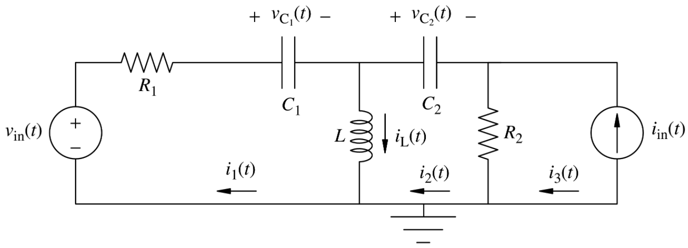

Figure 2: Electrical Circuit.

Using clockwise circulating mesh currents $i_1(t)$, $i_2(t)$, and $i_3(t)$, Kirchhoff's voltage and current laws yield:

$$R_1 i_1(t) + v_{C_1}(t) + L\frac{d}{dt}[i_1(t) - i_2(t)] = v_{in}(t),$$

$$L\frac{d}{dt}[i_2(t) - i_1(t)] + v_{C_2}(t) + R_2[i_2(t) - i_3(t)] = 0,$$

$$i_3(t) = -i_{in}(t), \qquad i_L(t) = i_1(t) - i_2(t).$$

Derive a valid state-space realisation for the electrical network. That is, specify the state variables and derive the coefficient matrices A, B, C, and D.

Question 3. *Consider the matrix*

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

(a) *Compute the characteristic polynomial of A and find the eigenvalues of A.*

(b) *Compute three linearly independent eigenvectors for A.*

(c) *Find a similarity transformation L such that $\hat{A} = L^{-1}AL$ is a diagonal matrix.*

(d) *Compute e^{At} as a function of t.*

(e) *Compute $\sin(At)$ as a function of t.*

Question 4. *Consider a linear continuous-time system described by the equations*

$$\begin{aligned} \dot{x}_1(t) &= x_1(t) + \alpha x_2 + u(t) \\ \dot{x}_2(t) &= x_1(t) + x_2(t) - \alpha x_2(t) \\ y(t) &= x_1(t) \end{aligned}$$

with $\alpha \in \mathbb{R}$ and constant, $x(t) = [x_1(t), x_2(t)]^T \in \mathbb{R}^2$ and $u(t) \in \mathbb{R}$.

(a) *Let $u(t) = 0$, for all $t \geq 0$. Compute the equilibrium points of the system as a function of α.*

(b) *Assume now $u(t) = u(0)$, for all $t \geq 0$, where $u(t) \neq 0$. Compute the equilibrium points of the system as a function of α.*

(c) *Discuss similarities and differences between the results in part (a) and part (b).*